Simultaneous Phase Unwrapping and Speckle Smoothing in SAR Images: A Stochastic Nonlinear Filtering Approach

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1 Abstract

The paper proposes a new method for simultaneous estimation of the absolute phase (not simply modulo-2π), the backsattering coefficients, and the correlation factor of a pair of complex *synthetic aperture radar* (SAR) images. We adopt a Bayesian framework. Therefore, we need the prior knowledge concerning the images to be restored (phase, backscattering coefficients, and correlation factors) and a suitable model for the observed image conditioned on the images to be restored. We assume as priors for the phase, the backscattering coefficients, and the correlation factors three independent *Markov random fields.* The observation model takes into account: the speckle noise of each SAR image, the decorrelation mechanisms (spatial and temporal), and the system electronic noise. Based on these models an iterative algorithm embodying a recursive stochastic nonlinear filter is derived. A set of experimental results illustrate the effectiveness of the proposed approach.

2 Introduction

SAR is a coherent system that produces high resolution images of the electric field backscatterd by the surface being illuminated [1]. SAR images are typically acquired by a single antenna. By using two antennas separated by a baseline B, it becomes possible to *interfere* the two images in such a way that the common scene reflectivity is canceled and the geometric information contained in the scene topography is retained in the phase difference. It is the so-called *interferometric synthetic aperture radar* (InSAR).

In a SAR system, as in any coherent system, only the principal values of the phase are available, as computed from the argument of the received wave. However, in InSAR applications, the objective is the estimation of the absolute phase, and not simply its modulo-2π.

Classical phase unwrapping methods are either of *path following type* or of least-squares type. In both cases, the absolute phase determination is usually based on the following two-step procedure (e.g., [2], [3], [4], [5] [6],

1. determine the so-called *wrapped-phase* image or *interferogram*, which is an estimate of the modulo-2π phase values;

2. based on the interferogram *unwrap* the phase (i.e., determine its absolute values) by adopting some heuristic or adhoc phase continuity criterion.

In the path following schemes, phase is unwrapped through selected image patches. In the presence of noise different patches between two points may lead to different absolute phase differences. Heuristic rules are applied to resolve or mitigate this inconsistencies [2], [7] Typically, unwrapping methods that do no rely on path following cast the problem in the least-squares formalism [8]. Recently, it has been shown that the least-squares solution to phase unwrapping is equivalent to the solution of the discretized Poisson equation with Newmann boundary conditions. This solution can be computed efficiently by using fast cosine or Fourier transforms [9].

Due to decorrelation (temporal and spatial) and no-return or low return areas, the modulo-2π phase estimates corresponding to those areas might be extremely noisy. In an attempt to include this information in the unwrapping procedures, the weighted least-squares approach has been used [10], taking as weights the correlation coefficients.

2.1 Proposed Approach

In this paper we propose a model based approach to phase unwrapping that does not share the spirit of the classical techniques. Absolute phase estimation is addressed from a statistical point of view: SAR images are described as random fields whose statistical properties are built upon the physical mechanism of image generation. The probability of this random fields is parametrized by the absolute phase, the backscattering coefficients, and the correlation factors. The Bayesian framework is adopted for the joint estimation of this image parameters.

In the Bayesian approach *a priori* probabilities of the fields to be estimated, supplies a suitable tool to model smoothness constraints on those fields. In this work prior knowledge is modelled by three independent *Gauss Markov random fields* (GMRF), one per parameter image.

Joint estimation of the three fields is implemented by an iterative scheme that, at each iteration, computes:

1. the joint estimation of the correlation and backsacttering coefficient fields, using a coordinate-wise ascent type algorithm;

2. the absolute phase image using a recursive nonlinear estimator.

It should be stressed that the estimates of the tree fields are interdependent according to the image generation model. This is in contrast with the unwrapping strategies, where each field is dealt with independently, in an adhoc fashion.

The recursive stochastic nonlinear filtering part of this work is on the vein of works [11], [12]. The problem therein addressed is the estimation of the absolute phase from noisy observations of its inphase and quadrature components. As shown latter, part of the estimation procedure herein considered, is cast exactly in this way.

3 Observation Model

For a given InSAR geometry, the terrain elevation is obtained from the phase $\phi = \phi_2 - \phi_1$ [13], where ϕ_1 and ϕ_2 are the propagation path phases read by the two antennas. Phase ϕ relates, in a noisy and nonlinear way, with the observed SAR images (interferometric pair).

Denote x_1 and x_2 as the complex amplitudes (inphase and quadrature components packed into complex numbers) of the backscattered field read by each antenna at a given pixel. These amplitudes are given by

$$\begin{aligned} x_1 &= z_1 e^{-j\phi_1} + n_1 && (1)\\ x_2 &= z_2 e^{-j\phi_2} + n_2, && (2)\end{aligned}$$

where z_i, $i = 1, 2$, is the complex amplitude originated by the scatterers illuminated by aperture i, and n_i is the respective electronic noise.

Assuming that the surface being illuminated is rough compared to the wavelength, that there are no strong specular reflectors, and that there are a *large* number of scatterers per resolution cell, then the complex amplitude z_i are circularly symmetric and Gaussian [14]. Noises n_i are independent of complex amplitudes z_i, and also circularly symmetric and Gaussian [15]. Complex amplitudes z_1 and z_2 are different due to spatial and temporal decorrelations. The former is originated by non-overlapping portions or the aperture regions of each antenna; the latter is originated by scatterer displacements. We assume that $E[|z_1|^2] = E[|z_2|^2] = \theta^2$ and that $E[z_1 z_2^*] \equiv \alpha\theta^2$, where α stands for the correlation factor between z_1 and z_2, also termed *change parameter* or *degree of coherence* [16]. We assume that $\alpha \in [0, 1]$, which is valid whenever the scatterer displacements have an even distribution.

Defining $x \equiv [x_1\ x_2]^T$, $E[|n_1|^2] \equiv \sigma_n^2$, and assuming that $E[|n_1|^2] = E[|n_2|^2]$, the probability density function of x is written as [16]

$$p(x|\phi, \theta, \alpha) = \frac{1}{\pi^2|\mathbf{Q}|} e^{-x^H \mathbf{Q}^{-1} x}, \qquad (3)$$

where $\mathbf{Q} \equiv E[xx^H]$ is given by

$$\mathbf{Q} = \begin{bmatrix} \theta^2 + \sigma_n^2 & \alpha\theta^2 e^{-j\phi} \\ \alpha\theta^2 e^{j\phi} & \theta^2 + \sigma_n^2 \end{bmatrix}. \qquad (4)$$

Let $x_{ij} \equiv [x_{1ij}\ x_{2ij}]^T$ denote the random vector of complex amplitudes associated to the pixel (i, j) and $\mathbf{x} \equiv \{x_{ij}, i, j = 1, \ldots, N\}$ (we assume without lack of generality that images are squared). Define $\boldsymbol{\phi} = \{\phi_{ij}, i, j = 1, \ldots, N\}$, $\boldsymbol{\theta} = \{\theta_{ij}, i, j = 1, \ldots, N\}$, and $\boldsymbol{\alpha} = \{\alpha_{ij}, i, j = 1, \ldots, N\}$. Assuming that the components of $\boldsymbol{x}$ are conditionally independent, then,

$$p(\mathbf{x}|\boldsymbol{\phi}, \boldsymbol{\theta}, \boldsymbol{\alpha}) = \prod_{ij=1}^{N} p(x_{ij}|\phi_{ij}, \theta_{ij}, \alpha_{ij}).$$

The conditional independence assumption is valid if the resolution cells associated to any pair of pixels are disjoint. Usually this is a good approximation, since the *point spread function* (PSF) of the SAR system is only slightly larger than the corresponding inter-pixel distance [17]. Anyway, the correlation introduced by the PSF can be modelled by assuming that the observed vector is $\mathbf{y} = \mathbf{B}\mathbf{x}$, where $\mathbf{B}$ is the associated blur matrix.

4 Prior Model

We assume that images $\boldsymbol{\theta}$, and $\boldsymbol{\alpha}$ are independent noncausal first order *Gauss Markov random fields* (GMRF) [18] with the following probability density functions:

$$\begin{aligned} p(\boldsymbol{\theta}) &= \frac{1}{Z_\theta} e^{-\mu_\theta \sum_{i,j=1}^{N} (\Delta\theta_{ij}^v)^2 + (\Delta\theta_{ij}^h)^2} && (5)\\ p(\boldsymbol{\alpha}) &= \frac{1}{Z_\alpha} e^{-\mu_\alpha \sum_{i,j=1}^{N} (\Delta\alpha_{ij}^v)^2 + (\Delta\alpha_{ij}^h)^2}, && (6)\end{aligned}$$

where $\Delta x^v_{ij} = (x_{i,j} - x_{i-1,j})$, $\Delta x^h_{ij} = (x_{i,j} - x_{i,j-1})$, Z_θ and Z_α are normalizing constants, and μ_θ and μ_α are smoothing controlling parameters. Priors (5) and (6) favor smooth fields. Estimates of $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$ result from the tradeoff between smoothness imposed by (5) and (6), and from data adjustment imposed by the observation mechanism trough pdf (3).

The image phase prior is modelled as an *autoregressive* (AR) process; specifically

$$\phi_{ij} = a_1\phi_{i,j-1} + a_2\phi_{i-1,j} + u_{ij}, \qquad (7)$$

where $\{u_{i,j}\}$ is a field of independent and identically distributed (i.i.d) zero mean Gaussian variables of variance σ_u^2. Parameters a_1 and a_2 are, typically set to 1/2, i.e., pixel $\phi_{i,j}$ is the mean of $\phi_{i,j-1}$ and $\phi_{i-1,j}$ plus a random independent increment. Moreover, prior $p(\boldsymbol{\phi})$ is also a first order GMRF with the same structure of (5) and (6).

The underlying reasons for adopting the AR model (7) are twofold:

1. smoothness enforcement on the phase $\boldsymbol{\phi}$ is possible by suitable choice of variance σ_u^2;
2. the pixel being updated depends only on those pixels already updated (past). In this scenario, a recursive stochastic filter can be applied to determine phase $\boldsymbol{\phi}$

The recursive stochastic filter demands a state-space description of (7). Herein, we adopt the *reduced order model* (ROM) proposed in [19], where the state vector contains only those pixels from which $\phi_{i,j+1}$ depends. Assuming the lexicographical order

$$\phi_{n(i,j)} \equiv \phi_{ij}, \qquad (8)$$

where $n(i,j) = iN + j$, the ROM for (7) is

$$\overbrace{\begin{bmatrix} \phi_{i,j+1} \\ \phi_{i-1,j+2} \end{bmatrix}}^{\mathbf{v}_{n+1}} = \overbrace{\begin{bmatrix} a_1 & a_2 \\ 0 & 0 \end{bmatrix}}^{\mathbf{A}} \overbrace{\begin{bmatrix} \phi_{i,j} \\ \phi_{i-1,j+1} \end{bmatrix}}^{\mathbf{v}_n} + \overbrace{\begin{bmatrix} \sigma_u \\ 0 \end{bmatrix}}^{\mathbf{B}} w_{i,j} + \overbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}^{\mathbf{E}} \overbrace{[\phi_{i-1,j+2}]}^{\mathbf{e}_n}, \qquad (9)$$

where the $w_{i,j}$'s are i.i.d unit variance zero mean complex Gaussian variables.

5 Estimation Procedure

Since the posterior distribution of $\boldsymbol{\phi}$, $\boldsymbol{\theta}$, and $\boldsymbol{\alpha}$, given the observed data $\mathbf{x}$, is known, one could think of computing the *maximum a posteriori probability* (MAP) estimates of those parameters according to

$$(\widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\alpha}})_{MAP} = \arg\max_{\boldsymbol{\phi},\boldsymbol{\theta},\boldsymbol{\alpha}} p(\boldsymbol{\phi}, \boldsymbol{\theta}, \boldsymbol{\alpha}|\mathbf{x}). \qquad (10)$$

However, this maximization would involve a high computational burdeen unbearable in most problems. The main reason for this is due to the observation mechanism, which only allows phase to be recovered modulo-2π. In this situation a recursive scheme is better suited than the batch one implicit in (10).

Aiming at the joint estimatation of $(\boldsymbol{\phi}, \boldsymbol{\theta}, \boldsymbol{\alpha})$, we propose an iterative procedure that at $(t+1)$-th iteration computes $(\widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\alpha}})^{(t+1)}$ and $\widehat{\boldsymbol{\phi}}^{(t+1)}$ according to

(1) Batch MAP Estimator:

$$(\widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\alpha}})^{(t+1)} = \arg\max_{\boldsymbol{\theta},\boldsymbol{\alpha}} p(\widehat{\boldsymbol{\phi}}^{(t)}, \boldsymbol{\theta}, \boldsymbol{\alpha}|\mathbf{x}) \qquad (11)$$

(2) Recursive Nonlinear Estimator:

$$\widehat{\boldsymbol{\phi}}^{(t+1)} = \{\widehat{\phi}_n^{(t+1)}, n = 1, \ldots, N^2\} : \qquad (12)$$

where ϕ_n is the first component of the state vector $\mathbf{v}_n$, modelled by (9), and

$$\widehat{\mathbf{v}}_n^{(t+1)} = \arg\max_{\mathbf{v}} p(\mathbf{v}|(\widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\alpha}})^{(t+1)}, \mathbf{x}_n) \qquad (13)$$

with $\{\mathbf{x}_n\} \equiv \{x_i,\, i = 1, \ldots, n\}$.

The iterative scheme is stopped when the difference $(\widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\alpha}})^{(t+1)} - (\widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\alpha}})^{(t)}$ reaches a preset threshold.

5.1 Batch MAP Estimator

The maximizer $(\widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\alpha}})^{(t+1)}$, expressed in (11), is implemented by a coordinate-wise ascent algorithm: the posterior distribution $p(\boldsymbol{\phi}^{(t)}, \boldsymbol{\theta}, \boldsymbol{\alpha}|\mathbf{x})$ is first maximized with respect to θ_n, for $n = 1, \ldots, N^2$, and next maximized with respect to α_n, for $n = 1, \ldots, N$. This two steps are repeated until the difference of consecutive estimates reaches a preset minimum value.

5.2 Recursive Nonlinear Estimator

To compute $\widehat{\mathbf{v}}_n$ according to (13), $p(\mathbf{v}_n|\mathbf{x}_n)$ must be known for all n ($(\widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\alpha}})^{(t+1)}$ is omitted for shortness). This is accomplished recursively in two steps:

Prediction

$$p(\mathbf{v}_n|\mathbf{x}_{n-1}) = \int p(\mathbf{v}_n|\mathbf{v}_{n-1}) p(\mathbf{v}_{n-1}|\mathbf{x}_{n-1})\, d\mathbf{v}_{n-1}, \qquad (14)$$

where $p(\mathbf{v}_n|\mathbf{v}_{n-1})$, reflecting (9), is Gaussian with mean $\mathbf{A}\mathbf{v}_{n-1} + \mathbf{E}\mathbf{e}_n$ and covariance matrix $\mathbf{B}\mathbf{B}^T$;

Filtering

$$p(\mathbf{v}_n|\mathbf{x}_n) \propto p(\mathbf{v}_n|\mathbf{x}_{n-1}) p(x_n|\mathbf{v}_n), \qquad (15)$$

where $p(x_n|\mathbf{v}_n) = p(x_n|\phi_n)$, which, according to observation model (3), takes the form

$$p(x_n|\phi_n) \propto \exp\{\lambda_n \cos(\phi_n - \eta_n)\}, \qquad (16)$$

with

$$\lambda_n = \frac{2\alpha_n|x_{1n}x_{2n}|}{\theta_n^2(1-\alpha_n^2)}, \qquad (17)$$

and

$$\eta_n = \arg\{x_{1n}^* x_{2n}\}. \qquad (18)$$

Note that λ_n, given by (17), has the meaning of a ratio between the correlated and the uncorrelated signal power.

5.2.1 Nonlinear Filter Implementation

To implement operations (14) and (15), finite representation operands are required. We adopt the strategy followed in [11], [12]. Essentially, it consists in representing the observation factor (16) as a train of Gaussian terms centered on $\eta_n^{(i)} \equiv \eta_n + 2\pi i$,for $i \in \mathcal{Z}$, and with a common variance γ_n. This variance minimizes the *Kulback distance* (defined in a 2π-interval), between $p(x_n|\phi_n)$ and the train of Gaussian terms. The relation between this optimal variance γ_n and the observation dependent parameter λ_n is provided by a look-up table of solutions of the above mentioned minimization. For details see [20].

Assume that $p(\mathbf{v}_n|\mathbf{x}_{n-1})$ is Gaussian; straight application of the filtering step would produce an infinite number of Gaussian terms. We adopt the following simplification: in multiplication (15) take only the term of the Gaussian sum representation closest to $E\{\phi_{1n}|\mathbf{x}_{n-1}\}$. The resulting density $p(\mathbf{v}_n|\mathbf{x}_n)$ turns out to be also Gaussian, the mean of which is taken as the optimal (MAP) estimate $\widehat{\phi}_{1n}$. After this, prediction (14) is now implemented as the usual Kalman-Bucy prediction step [21].

6 Experimental Results

Figure 1 displays two Gaussian phase elevations to be estimated from an InSAR image pair. The maximum phase variation between two grid points is 0.35 rad. On the left elevation the signal power and the correlation coefficient is set to $\theta = 1$ and $\alpha = 0.99$, respectively. On the right elevation, this parameters are set to $\theta = 10$ and $\alpha = 0.7$, respectively. The independent noise is taken to be zero, since the noise due to decorrelation is, typicaly, the most serious degradation effect in InSAR.

Figures 2 and 3 shows the interferogram $\boldsymbol{\eta}$ and the $\boldsymbol{\lambda}$-image as given by expression (17). As expected, the interferogram correspondent to the right elevation is much noisy than the interferogram correspondent to the left elevation. The same pattern is observed in the λ-image displayed in Fig. 3.

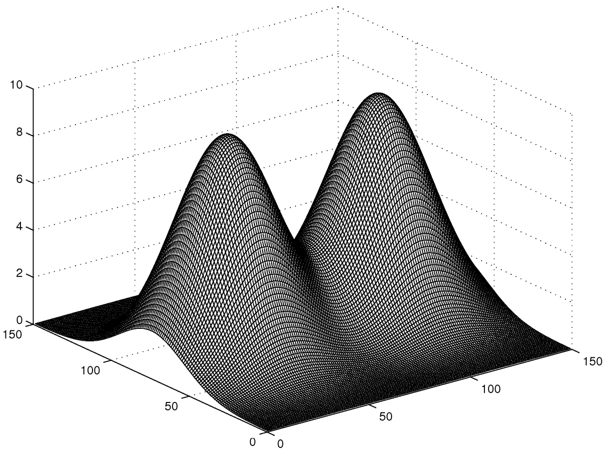

Figure 1: Original phase image.

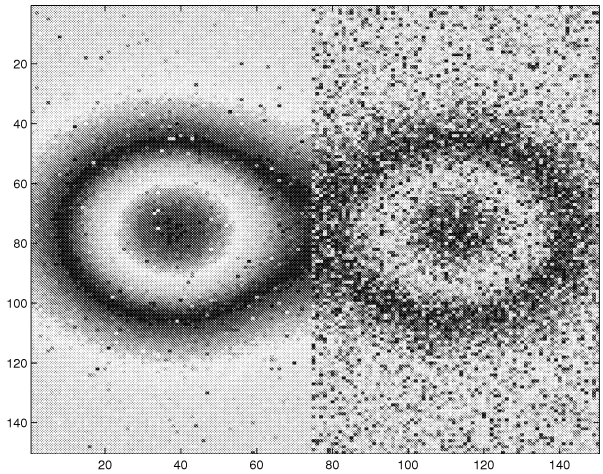

Figure 2: Interferometric image η.

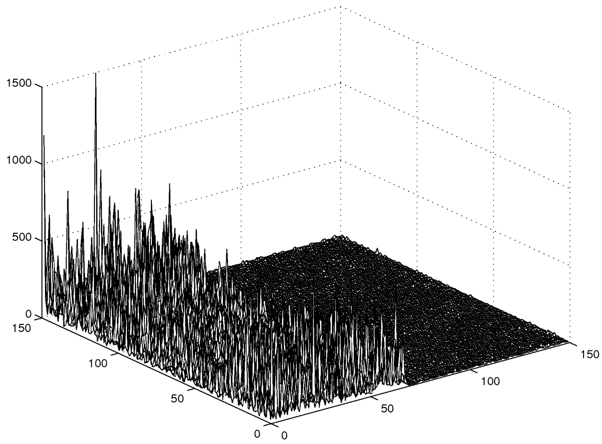

Figure 3: Equivalent sample signal-to-noise ratio λ-image.

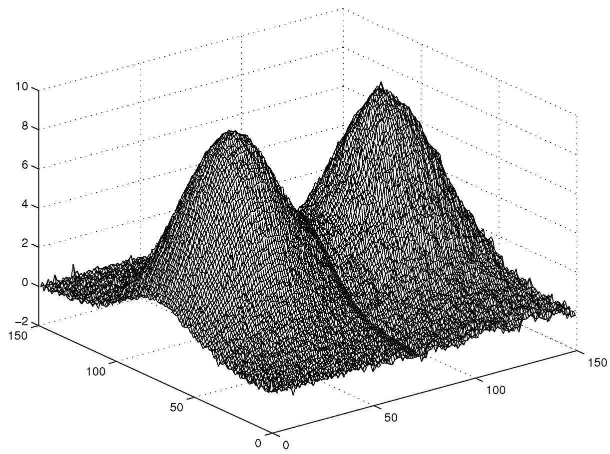

Figure 4: Estimated phase absolute image.

Finally 4 exhibits the correctly unwrapped phase on both elevations. The variance of the estimates on the left is, as it would be expectable, smaller.

7 Concluding Remarks

The paper addressed the problem of InSAR absolute phase image estimation from a Bayesian model-based point of view. The adopted observation model reflects the physics of the InSAR problem, whereas the prior models allow smoothness enforcement.

Not only the phase, but also the backscattering coefficients and the correlation factors are parameters of the overall model. As a by-product of phase unwrapping, the proposed algorithm also provides estimated images of the other two fields.

The goal of this work is to develop estimation methodologies independent, as much as possible, of adhoc procedures.

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