

Adaptive Restoration of Speckled SAR Images*

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June 17, 2007

Abstract

The paper proposes a restoration method for speckled images generated by coherent imaging systems (e.g., synthetic aperture radar, synthetic aperture sonar, ultrasound imaging, and laser imaging). These systems are invariably affected by *speckle noise* and therefore restoration/filtering of the mean backscattered signal (*backscattering coefficient*) is often necessary. The approach is Bayesian: the observed image is assumed to be a realization of a random field built upon the physical mechanism of image generation; the backscattering coefficient image is modelled by a *compound Gauss-Markov random field* which enforces smoothness on homogeneous regions while preserving discontinuities between neighboring regions. The *maximum a posteriori probability* (MAP) criterion is adopted. An *expectation maximization* type iterative scheme embedded in a continuation algorithm is used to compute the MAP solution. Application examples performed on radar real and synthetic data are presented.

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1 Introduction

Coherent imaging systems are designed aiming at the acquisition of the scene *complex reflectivity*. They are linear systems whose output is given by the convolution between its coherent *point spread function* (PSF) and the scene complex reflectivity. Examples are *synthetic aperture radar* (SAR), *synthetic aperture sonar* (SAS), *ultrasound imaging*, and laser imaging. The complex reflectivity originated in a given *resolution cell* is composed by the contributions of all individual scatterers lying in that cell. These contributions interfere randomly in a destructive or constructive manner, according to the spatial configuration of the scatterers. This random fluctuation is termed (*speckle noise*); its statistical properties has been addressed in several references [1], [2], [3]. Assuming that the surface being illuminated is rough compared to the wavelength, that there are no strong specular reflectors, and that there is a large number of scatterers per resolution cell, then the squared amplitude (*intensity*) of the complex reflectivity is exponentially distributed; the scenario just described, termed *fully developed speckle*, leads to highly noisy intensity images: the *signal to noise ratio* (SNR), defined as the square of the ratio between

the intensity mean value (backscattering coefficient) and the intensity variance, is one. The granular appearance of intensity images is due to this very low SNR.

Most applications involving coherent systems data rely on the mean intensity image (backscattering coefficient image). In view of the rationale above presented, there is need for applying speckle reduction/restoration techniques to intensity data. A common approach consists in averaging independent observations of the same pixel, which in the case of SAR systems is called *multi-look*. This term stems from the fact that each independent sample is generated by a different segment of the SAR array. Independently of the system, an image formed by the averaging of m independent samples will herein be termed an m -look image. For fully developed speckle, the SNR of an m -look image is m . However, increasing the number of independent samples results in reduction of spatial resolution, being necessary to resort to spatial smoothing techniques. The basic idea underlying these techniques is that of applying nonuniform smoothness in such a way that homogeneous image regions, in a statistical sense, are highly smooth, while discontinuities are preserved [4], [5], [6].

In recent years a significant research activity has been devoted to the development of speckle reduction techniques, or, equivalently, to the mean backscattering coefficient estimation. These techniques take the form of image restoration [7], [8], edge detection [9], or image segmentation algorithms [3], [10], [11], [12]. A common assumption is that images are locally smooth. This behavior has been modeled mainly by *ad hoc* techniques [7], or by *Markov random fields* [3], [10], [11], [12].

The present approach is Bayesian:

- the observed image, given the backscattering coefficients, is assumed to be a realization of a random field taking into account the statistics of image reflectivity;
- the set of backscattering coefficients associated to the image pixels is assumed to be *piecewise smooth*, and modelled as a random field with a *compound Gauss-Markov random field* (CGMRF) prior [5], [13].

The piecewise smoothness assumption makes sense in many situations: e.g., SAR images of agricultural landscapes, echography of the human body, etc.

To our knowledge, modeling the backscattering coefficient image with a CGMRF has not yet been addressed in the speckled image restoration context. and Tiago Silva

*This work was supported by Portuguese PRAXIS XXI program, under project 2/2.1.TIT/1580/95

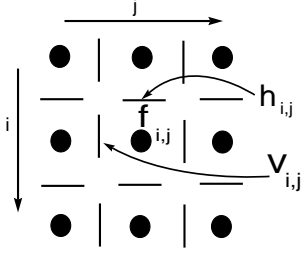


Figure 1: Relative positions of grids for the \mathbf{f} , \mathbf{v} , and \mathbf{h} fields.

2 Proposed Approach

Assume that images are defined in the $N \times N$ rectangular lattice $Z_N = \{(i, j), i, j = 1, \dots, N\}$. Define $\mathbf{F} = \{F_{ij}\}$ and $\mathbf{G} = \{G_{ij}\}$, with $(i, j) \in Z_N$ as the random fields associated to the backscattering coefficient image and the observed intensity image, respectively. Lowercase letters will denote the values assumed by the the random fields as well as its realizations. For compactness, the probability density of the generic field \mathbf{X} , $p_{\mathbf{x}}(\mathbf{X} = \mathbf{x})$, or of the generic random variable X , $p_x(X = x)$, will be denoted by $p(\mathbf{x})$ and $p(x)$, respectively.

Besides fields \mathbf{F} and \mathbf{G} , it is assumed the existence of another field, $\mathbf{L} = \{\mathbf{H}, \mathbf{V}\}$, with $\mathbf{H} = \{H_{ij}\}$ and $\mathbf{V} = \{V_{ij}\}$, signaling horizontal and vertical discontinuities. Variables V_{ij} and H_{ij} are binary, taking value 1 if a discontinuity is present and 0 otherwise. The field \mathbf{L} also termed *line field* [5] serves the purpose of avoiding edges to be smoothed out during the restoration of field \mathbf{F} . Fig. 1 schematizes the spatial arrangement of \mathbf{F} and \mathbf{L} .

The present work aims at the estimation of the backscattering coefficient, \mathbf{F} , and the line field, \mathbf{L} , from the observed image \mathbf{G} . Since our approach is Bayesian it is model-based, therefore not only the the image generation mechanism $p(\mathbf{g}|\mathbf{f}, \mathbf{l})$, but also the prior density $p(\mathbf{f}, \mathbf{l})$, which captures the model, must be specified.

2.1 Image Generation Mechanism

Under the fully developed speckle hypothesis, the complex amplitude $x = x_r + jx_i$ (inphase and quadrature components) of the backscattered field, at each pixel, is circularly symmetric and Gaussian [10]. Thus

$$p(x|f) = \frac{1}{\pi f^2} e^{-\frac{|x|^2}{f^2}}, \quad (1)$$

where $f^2 := E[|x|^2]$ is the backscattering coefficient of the referred resolution cell. For intensity or power images the data is in the form of square magnitude of the complex components, $g = |x_r + jx_i|^2$. Random variable G is therefore the exponentially distributed:

$$p(g|f) = \frac{1}{f^2} e^{-\frac{g}{f^2}}. \quad (2)$$

For an m -look image, G is the average of m independent exponentially distributed random variables, thus having a gamma density [14],

$$p(g|f) = \frac{1}{\Gamma(M)} \left(\frac{f^2}{M}\right)^{-M} g^{M-1} \exp\left(-\frac{gM}{f^2}\right), \quad (3)$$

with $E[g|f] = f^2$ and $\sigma^2[g|f] = f^4/M$.

It is herein assumed that the components of \mathbf{g} given \mathbf{f} are independent. Hence

$$p(\mathbf{g}|\mathbf{f}) = \prod_{ij \in Z_N} p(g_{ij}|f_{ij}). \quad (4)$$

The conditional independence assumption is valid if the resolution cells associated to any pair of pixels are disjoint. This is only true if the size of the imaging system PSF is smaller than the corresponding interpixel distance. This is approximately true in most acquisition systems. Otherwise, neighboring data would be extremely correlated adding no information.

2.2 Prior Model

Image \mathbf{f} is assumed to be *piecewise smooth*. This makes sense whenever the scene is made of smooth regions, concerning the backscattering coefficient. *Gauss-Markov random fields* [15] are both mathematically and computationally suitable for representing local interactions, and particularly continuity between neighboring pixels. However, the continuity constraint must be discarded for those pixels near the discontinuities. For this purpose we take the *first order noncausal CGMRF*

$$p(\mathbf{f}|\mathbf{l}) \propto \exp\left\{-\frac{\mu}{2} \sum_{ij} (\Delta_{ij}^h)^2 \bar{v}_{ij} + (\Delta_{ij}^v)^2 \bar{h}_{ij}\right\}, \quad (5)$$

where $\bar{v}_{ij} := (1 - v_{ij})$, $\bar{h}_{ij} := (1 - h_{ij})$, $\Delta_{ij}^h := (f_{ij} - f_{i,j-1})$, $\Delta_{ij}^v := (f_{ij} - f_{i-1,j})$, and μ^{-1} has the meaning of the variance of the increments Δ_{ij}^h and Δ_{ij}^v . Notice that continuity constraint between pixels (i, j) and $(i, j - 1)$ is removed if variable v_{ij} is set to one; the same is true concerning horizontal lines. Model (5) is not, however, normalizable and therefore it is not a probability density function. To solve this problem, the following prior was proposed in [13]:

$$p(\mathbf{f}|\mathbf{l}) = \frac{1}{Z(\mathbf{l})} e^{-\frac{\mu}{2} \sum_{ij} \omega (\Delta_{ij}^h)^2 \bar{v}_{ij} + \omega (\Delta_{ij}^v)^2 \bar{h}_{ij} + (1-4\omega) f_{ij}^2}, \quad (6)$$

where ω is the discontinuity smoothness parameter, which should be chosen so as to $\omega < 1/4$, and $Z(\mathbf{l})$ is a normalizing constant depending on \mathbf{l} , termed *partition function*. For values of ω very close to 1/4 the third term of (6) affects very little the result, yet leads to an integrable prior $p(\mathbf{f}|\mathbf{l})$.

Apart a constant, the inverse of the partition function $Z(\mathbf{l})$ is given by the determinant of the $N^2 \times N^2$ -matrix associated to the quadratic form present in (6). This is an unbearable task due to the typically huge size of the matrix involved and also due to its irregularity associated to the particular discontinuity pattern. To circumvent the difficulty therein, the density $p(\mathbf{f}|\mathbf{l})$ is replaced by the Besag's pseudolikelihood approximation [15]. Defining $f_{N_{ij}} := \{f_{i,j-1}, f_{i,j+1}, f_{i-1,j}, f_{i+1,j}\}$, $l_{N_{ij}} := \{h_{i,j}, v_{i,j}, h_{i+1,j}, v_{i,j+1}\}$, $f_{ij} := \bar{h}_{ij} f_{i-1,j} + \bar{h}_{i+1,j} f_{i+1,j} + \bar{v}_{ij} f_{i,j-1} + \bar{v}_{i,j+1} f_{i,j+1}$, and

$$\eta_{ij} := \lambda_{ij}^2 \mu \omega \bar{f}_{ij} \quad (7)$$

$$\lambda_{ij}^2 := \frac{1}{\mu [1 - \omega(v_{ij} + v_{i,j+1} + h_{ij} + h_{i+1,j})]}, \quad (8)$$

the pseudolikelihood approximation is written as (see [16])

$$\begin{aligned}
p(\mathbf{f}|\mathbf{l}) &\simeq \prod_{ij} p(f_{i,j}|f_{N_{i,j}}, l_{N_{i,j}}) \\
&= \prod_{ij} \frac{1}{\sqrt{2\pi}\lambda_{ij}} \exp\left\{-\frac{1}{2\lambda_{ij}^2}(f_{i,j} - \eta_{ij})^2\right\} \quad (9) \\
&= \frac{e^{-\mu \sum_{ij} \omega(\Delta_{ij}^h)^2 \bar{v}_{ij} + \omega(\Delta_{ij}^v)^2 \bar{h}_{ij} + \frac{1}{2}(1-4\omega)f_{ij}^2}}{\prod_{ij} \sqrt{2\pi}\lambda_{ij}}, \quad (10)
\end{aligned}$$

where expression (10) is obtained from (9) after a simple but lengthy manipulation.

The probability $p(\mathbf{l})$ is chosen, as in the *weak membrane* [4] model, to add a penalty to each discontinuity signaled:

$$p(\mathbf{l}) = \frac{1}{Z_\alpha} e^{-\alpha \|\mathbf{l}\|}, \quad \alpha > 0, \quad (11)$$

where $\|\mathbf{l}\| := \sum_{ij} (v_{ij} + h_{ij})$. Prior (11) is only a function of the number of discontinuities signaled.

2.3 Joint Density

Invoking the Bayes rule, and noting that $p(\mathbf{g}|\mathbf{f}, \mathbf{l}) = p(\mathbf{g}|\mathbf{f})$, we obtain the joint probability density function of $(\mathbf{f}, \mathbf{l}, \mathbf{g})$ as

$$p(\mathbf{f}, \mathbf{l}, \mathbf{g}) = p(\mathbf{g}|\mathbf{f})p(\mathbf{f}|\mathbf{l})p(\mathbf{l}). \quad (12)$$

Replacing expressions (3), (10), and (11) in (12), the joint distribution of \mathbf{f}, \mathbf{l} conditioned on \mathbf{g} , is then given by

$$p(\mathbf{f}, \mathbf{l}|\mathbf{g}) = \frac{1}{Z} e^{-U(\mathbf{f}, \mathbf{l}|\mathbf{g})}, \quad (13)$$

where

$$\begin{aligned}
-U(\mathbf{f}, \mathbf{l}|\mathbf{g}) &= \quad (14) \\
&- 2M \log f_{ij} - \frac{Mg_{ij}}{f_{ij}^2} \\
&- \mu \sum_{ij} \omega \bar{v}_{ij} (\Delta_{ij}^h)^2 + \omega \bar{h}_{ij} (\Delta_{ij}^v)^2 + \frac{1}{2}(1-4\omega)f_{ij}^2 \\
&- \sum_{ij} \log \lambda_{ij} - \log Z_\alpha - \alpha \|\mathbf{l}\| + c^{te}. \quad (15)
\end{aligned}$$

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3 Computing the MAP solution

The *maximum a posteriori probability* (MAP) criterion is adopted for estimating both \mathbf{f} and \mathbf{l} . Accordingly

$$(\hat{\mathbf{f}}, \hat{\mathbf{l}})_{MAP} = \arg \max_{\mathbf{f}, \mathbf{l}} p(\mathbf{f}, \mathbf{l}|\mathbf{g}). \quad (16)$$

Computing the MAP solution leads to a huge non-convex optimization problem, involving continuous and discrete variables, with unbearable computation burden. Instead of determining exactly $(\hat{\mathbf{f}}, \hat{\mathbf{l}})_{MAP}$, we propose a continuation method which although not

yielding the global maximum of $p(\mathbf{f}, \mathbf{l}|\mathbf{g})$, delivers nearly optimum estimates with a feasible computational load. Aiming at this goal, define

$$p(\mathbf{f}, \mathbf{l}|\mathbf{g}, \beta) = \frac{1}{Z(\beta)} e^{-\beta U(\mathbf{f}, \mathbf{l}|\mathbf{g})}. \quad (17)$$

Under a statistical physical interpretation, parameter β is the inverse of the temperature ($\beta = 1/T$); this parameter controls the prominence of the maxima of (17): when $\beta \rightarrow 0$, all configurations of (\mathbf{f}, \mathbf{l}) are equiprobable; when $\beta \rightarrow \infty$, the absolute maxima becomes progressively more marked, and in limit the set of the absolute maxima have probability one.

Annealing algorithms, be it stochastic or deterministic, exploit this behaviour to establish continuation methods in which the temperature plays the role of continuation parameter.

The proposed continuation scheme evaluates, for an increasing sequence β_t , with $t = 1, \dots, t_m$, the maximum $\hat{\mathbf{f}}^{(t)}$ of $p(\mathbf{f}, \hat{\mathbf{l}}^{(t-1)}|\mathbf{g})$, with respect to \mathbf{f} , followed by the *mean line field* $\hat{\mathbf{l}}^{(t)} = E_{\beta_t}[\mathbf{l}|\hat{\mathbf{f}}^{(t)}, \mathbf{g}]$ (symbol E_β denotes the mean value operator computed according to density (17)). Based on this operation the algorithm is termed *expectation maximization annealing* (EMA).

EMA Algorithm

Initialization: set $\hat{h}_{ij}^{(0)} = \hat{v}_{ij}^{(0)} := 0.5$, $t = 1$, β_0 , a , m
DO

step 1: $\hat{\mathbf{f}}^{(t)} = \arg \max_{\mathbf{f}} p(\mathbf{f}, \hat{\mathbf{l}}^{(t-1)}|\mathbf{g})$ (18)

step 2: $\hat{\mathbf{l}}^{(t)} = E_{\beta_t}[\mathbf{l}|\hat{\mathbf{f}}^{(t)}, \mathbf{g}]$ (19)

step 3: $\beta_{t+1} = a\beta_t$ (20)

While $t \leq m$

Due to the the already pointed out behaviour of (17), it follows that

$$E_\beta[\mathbf{l}|\mathbf{f}, \mathbf{g}] \rightarrow \arg \max_{\mathbf{l}} p(\mathbf{f}, \mathbf{l}|\mathbf{g}), \quad \beta \rightarrow \infty. \quad (21)$$

Therefore the stationaty points of the EMA algorithm are, at least, local maxima of $p(\mathbf{f}, \mathbf{l}|\mathbf{g})$; the *quality* of the maxima depends on the schedule of β_t .

Maximization (18) is implemented by the *iterated conditional modes* (ICM) method [15]. This is an coordinate-wise ascent technique, that maximizes the posterior distribution with respect to each individual component. After sweeping all image, the procedure is repeated until no noticeable energy increments are obtained.

The sationary points of (15), with respect to f_{ij} , are zeros of the fourth order polynomial

$$\begin{aligned}
&-\mu \left((1-4\omega) + 2\omega(\bar{h}_{ij} + \bar{h}_{i+1,j} + \bar{v}_{ij} + \bar{v}_{i,j+1}) \right) f_{ij}^4 \\
&+ 2\mu\omega \bar{f}_{ij} f_{ij}^3 - 2M f_{ij} + 2Mg_{ij} = 0. \quad (22)
\end{aligned}$$

For each pixel (i, j) the solution of equation (22) is chosen to be the one root that maximizes $p(\mathbf{f}, \hat{\mathbf{l}}^{(t-1)}|\mathbf{g})$.

Step (19) of the EMA algorithm is similar to the equivalent step of *mean field algorithm* (MFA); using the rationale proposed in [6], one obtains

$$v_{ij} := E_{\beta}[v_{ij}|\mathbf{f}, \mathbf{g}] \simeq \frac{1}{1 + e^{\beta\{\alpha_{ij}^v - \mu\omega(\Delta_{ij}^h)^2\}}}, \quad (23)$$

where $\alpha_{ij}^v = \alpha + \frac{1}{2} \log t_{ij}^v$ and (see [16])

$$t_{ij}^v = \frac{[1 - \omega(v_{i,j-1} + h_{i,j-1} + h_{i+1,j-1})]}{[1 - \omega(1 + v_{i,j-1} + h_{i,j-1} + h_{i+1,j-1})]} \times \frac{[1 - \omega(v_{i,j+1} + h_{ij} + h_{i+1,j})]}{[1 - \omega(1 + v_{i,j+1} + h_{ij} + h_{i+1,j})]}. \quad (24)$$

A similar result is obtained for $\bar{h}_{ij} := E_{\beta}[h_{ij}|\mathbf{f}, \mathbf{g}]$, replacing h by v and vice versa (23) and (24).

The EMA algorithm differs from the MFA one in the step (19): while the latter evaluates mean value $E_{\beta_i}[\mathbf{f}|\hat{\mathbf{I}}^{(t)}, \mathbf{g}]$, the former maximizes $p(\mathbf{f}, \hat{\mathbf{I}}^{(t)}|\mathbf{g})$ with respect to \mathbf{f} . For Gaussian observation models both estimates are equal. This is not, however, the present case, since the observation model is not Gaussian.

Although not explicitly, function $p(\mathbf{f}, \mathbf{I}|\mathbf{g})$ depends on parameters μ and α . The maximum likelihood estimate of μ is

$$\hat{\mu}_{ML} = \frac{N^2}{2 \sum_{ij} \omega \bar{v}_{ij} (\Delta_{ij}^h)^2 + \omega \bar{h}_{ij} (\Delta_{ij}^v)^2 + \frac{1}{2}(1 - 4\omega) f_{ij}^2}. \quad (25)$$

Therefore, if μ is unknown, it can be iteratively evaluated when implementing the EMA.

The setting of parameter α , depends on the thresholding above which discontinuities are signaled. This can be understood from expression (23), which for high values of β leads to the following decision rule:

$$v_{ij} = \begin{cases} 1, & (f_{ij} - f_{i,j-1})^2 > \frac{\alpha + \frac{1}{2} \log t_{ij}^v}{\mu\omega} \\ 0, & (f_{ij} - f_{i,j-1})^2 < \frac{\alpha + \frac{1}{2} \log t_{ij}^v}{\mu\omega}. \end{cases} \quad (26)$$

Assume that the discontinuities from which t_{ij} depends are all set to zero. In this case we have $\frac{1}{2} \log t_{ij}^v = 0.29$, which typically verifies $\alpha \gg \log t_{ij}^v$; a vertical discontinuity is therefore signaled if $\Delta_{ij}^h > \sqrt{\alpha/(\mu\omega)}$. If some discontinuities from which t_{ij} depends are set to one, the threshold $\frac{\alpha + \frac{1}{2} \log t_{ij}^v}{\mu\omega}$ increases, preventing an over segmented image, namely the creation of double edges.

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4 Experimental Results

In this section we present restoration results applied to synthetic and real data. The EMA algorithm is parametrized with $m = 10$ (number of iterations), and $a = 1.259$ (increasing rate of β). Parameter μ is estimated according to (25).

Fig. 2 shows a restoration example of a synthetic image. The different image parts show:

(a,b) the original parameter image comprising 4 constant regions;

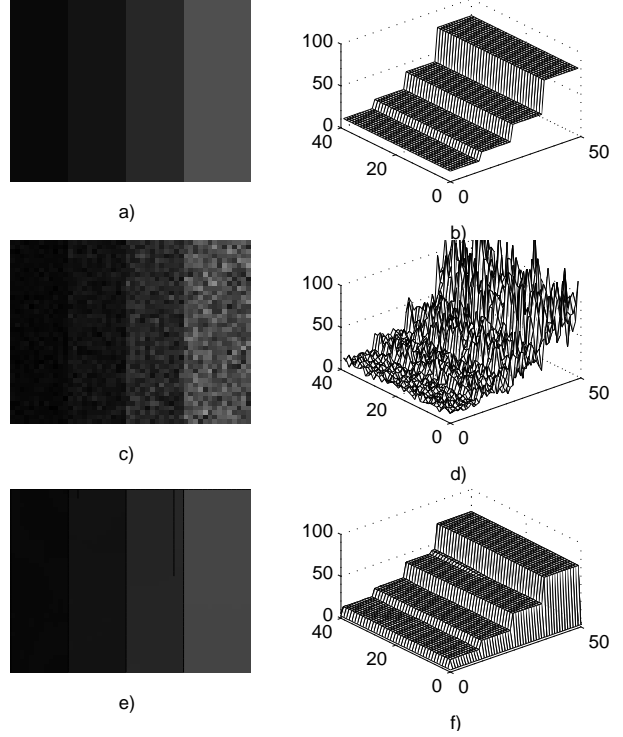


Figure 2: Synthetic image: a) and b) original image; c) and d) square root of noisy images generated for 3-looks; e) and f) restored image. In e) the resulting segmentation is superimposed.

(c,d) square root of the gamma image generated using the previous image as parameter with $M = 3$ (number of looks);

(e,f) the restored image.

Both images are showed as colormaps and surfaces for better perception. In the estimated image, the estimated discontinuities are overimposed. The estimated image exhibits nearly perfectly preserved edges. Within each region the image is smooth as the original image and the estimated values are very close to the original. Although the estimation was based on a very noisy image (*signal-to-noise ratio* of 4.7 dB) the results were expectable due to the fact that the image complies totally with the CGMRF prior.

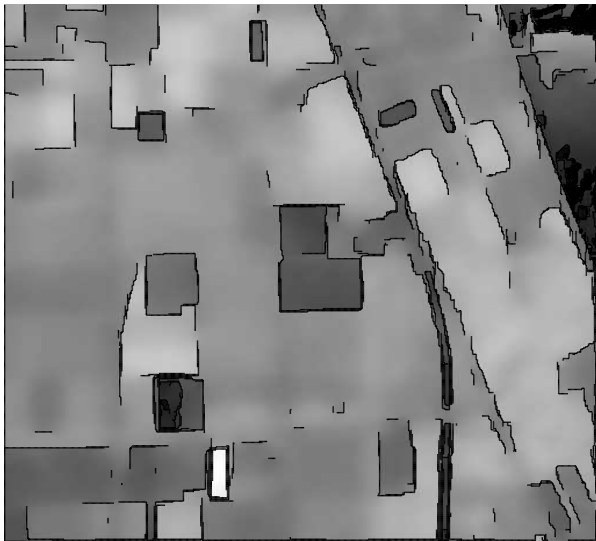
Fig. 3(a) shows a SAR image of the agricultural landscape of Flevoland region in northern Netherlands. The image was captured in Radarsat's standard mode (pixel spacing $\simeq 12.5\text{m}$) and has four-looks. Part (b) displays the restored image with the line field superimposed. The EMA algorithm performs very well, as it can be perceived from the plot shown in part (c) of Fig. 3: the squared root of intensity data jointly with the restored image are plotted for column 300 (image has size 512×512). The algorithm correctly smooths data within homogeneous regions, preserving discontinuities between neighboring regions. and Tiago Silva

5 Concluding Remarks

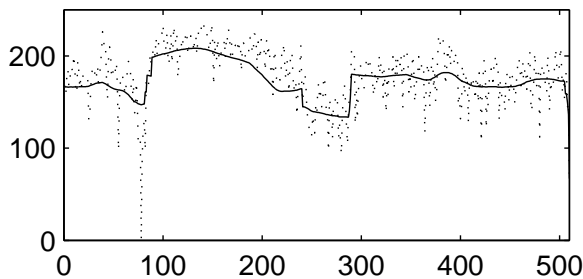
A new method for edge-preserving restoration of coherent images was presented. The knowledge of the statistics of the speckled im-



a)



b)



c)

Figure 3: Radarsat SAR image of agricultural fields in the Flevoland, Netherlands: a) original image; b) estimated image considering $M = 4$. The resulting segmentation is superimposed on both a) and b); estimated values along a straight line.

ages was fully taken into account through the observation model. The prior, a *compound Gauss Markov random field*, was used to tackle images exhibiting piecewise homogenous regions.

Restoration examples, comprising very noisy synthetic and SAR images, suggest the adequacy of the proposed methodology. Estimation of the model's parameters, will be further improved in order to render the algorithm more robust and automatic.

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