Nonlinear Stochastic Filtering Technique for Radar/Lidar Inversion

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ABSTRACT

The paper addresses the joint estimation of backscatter and extinction coefficients from range/time noisy data under a nonlinear stochastic filtering setup. This problem is representative of many remote sensing applications such as weather radar and elastic-backscatter lidar. A Bayesian perspective is adopted. Thus, in addition to the observation mechanism, relating in a probabilistic sense the observed data with the parameters to be estimated, a prior probability density function has to be specified. We adopt as prior a causal first order auto-regressive (AR) Gauss-Markov random field (GMRF). By using a reduced order state-space representation of the prior, we derive a nonlinear stochastic filter that recursively computes the backscatter and extinction coefficients at each site. A set of experiments based on simulated data illustrates the potential of the proposed approach.

Keywords: signal inversion, stochastic filtering, radar, lidar, parameter estimation

1. INTRODUCTION

An important class of remote sensing applications aims at the measurement of the backscatter and/or extinction coefficients in the space volume being scanned. Relevant examples are weather radar[1,2] and elastic-backscatter lidar,[3] .[4,5]

A wave propagating in a random media suffers attenuation and scattering. Under the *first order multiple scattering* approach,[6] the multiple scattering among scatters is negligible. The mean power received by the transducer is, under this hypothesis, given by

$$P(z) = \frac{C}{z^2}\,\beta(z)\exp\left(-2\int_0^z \alpha(x)\,dx\right), \tag{1}$$

where $P(z)$ (W) is the mean power received from range z (km), C (W km^3) is a system dependent constant, and β (km^{-1} sr^{-1}) and α (km^{-1}) are the backscatter and extinction coefficients (herein sometimes termed *clutter parameters*).

Signals acquired by remote sensing systems are not mean values; instead, they are random variables whose mean values are given by (1). Therefore, any inversion scheme aimed at the computation of the clutter parameters has to do so based on noisy and nonlinear observations. Moreover, the clutter is typically nonhomogeneous and nonstationary leading to time varying parameters α and β. This characteristic introduces additional complexity in the signal/image processing scheme, since the size of data available to determine each pair (α, β) is limited.

1.1. Classical Approaches

Relevant techniques for the inversion of clutter parameters are the Klett's method,[3] the slope and exponential fitting methods (see,[7] [4]), and the extended Kalman filtering approach.[8]

The Klett method assumes that to the mean power $P(z)$ (more precisely $S(z) = \log(P(z)z^2)$) is available and that the constitutive relation between clutter parameters is known and is of the form $\beta(z) = B_0\alpha(z)^b$, where B_0 and b are constants. Under this circumstances, Klett derived the exact solution for the attenuation coefficient $\alpha(z)$.

The major weakness of the Klett's method is that it is a deterministic approach that relies on the mean power P. Typically, only a noisy version of P is available, leading to numerically unstable solutions. This problem is attenuated by adopting a backward integration scheme that computes the extinction coefficient based on its guess at the farthest range of the inversion.

The exponential fitting technique applies to homogeneous clutter, i.e., the backscatter and extinction coefficients are assumed constant; the clutter parameters estimates are found by minimizing the the least square error between the observed data and P given by (1). The slope method is also a least-square fitting technique, but applied to $S(z) = \log(P(z)z^2)$. The estimates provided by the exponential fitting technique, despite being computationally more demanding (its is a nonlinear fitting problem), are better than those given by the slope method.[7]

For homogeneous clutter, the slope and exponential fitting methods are robust in the sense that they do not assume any prior constitutive relation and deal with noisy data. They have therefore advantage over the Klett's approach. However, the homogeneous assumption severely limits the application scope of this methodologies.

Work[8] proposes an extended Kalman filter that in each step estimates a vector of clutter parameters associated to a given range interval. The state vector equation is a first order auto-regressive vector process; basically, it imposes, in a probabilistic sense, smoothness between two consecutive time estimates at a given range. The smoothness between components within the state vector is established by imposing nonzero correlation among these components.

The approach proposed in[8] embodies the better features of the klett's and of the least square fitting procedures: it does not assume homogeneous clutter, it deals with the noisy observations, and, by imposing correlation on the clutter parameters, it allows to include prior information about the constitutive relation in a probabilistic sense.

In our opinion the drawbacks of the extended Kalman filtering approach proposed in[8] are the robustness of estimates with respect to state space noise correlation matrix and its complexity, in the computational sense. For example, if the clutter parameters are to be estimated within a a range size of 5Km with a resolution of 20m, the implementation of the EKF involves operations with matrices of size 500×500.

1.2. Proposed Approach

Assume that the observed data and the clutter parameters are arranged into 2D fields (images); data along rows is associated to the echo of a given time pulse (time coordinate), whereas data along columns is associated to a given range (range coordinate).

We propose a stochastic nonlinear filtering (NLF) solution to clutter parameter estimation. Being a stochastic filtering approach, it relies on the observed data probability density function (p.d.f.), given the parameters, and the space-state equation:

1. Observed data images are described through the respective p.d.f. of the observed data given the clutter parameters. In this work we consider only the Gaussian p.d.f., which models, for example, large aperture lidar applications.[9] The methodology is however easily adapted to other statistics, such as the exponential and Gamma, covering a large range of scenarios in weather radar and lidar.
2. The space-state equation is taken as the *reduced order model* (ROM)[10] description of a causal first order auto-regressive (AR) Gauss-Markov random Field (GMRF). For this to be possible we use as state variables the backscatter coefficient and the along-path integrated extinction coefficient (integrated extinction).

We stress that stochastic filtering is a Bayesian approach where the state space equation plays the role of prior. The type of prior knowledge that we intend to describe is that of smoothness between neighboring parameters. This is attained by properly choosing the regression parameters of the AR model.

The NLF solution we propose is a recursive scheme that propagates from one site to the next site, in a lexicographical order, the probability of the clutter parameters conditioned to past observed data (the so-called filtering density).

1.3. Paper Overview

The paper is organized as follows: the next section introduces formally the clutter parameter estimation and present the observation mechanism, the prior model, and the ROM representation. Section 3 elaborates on the NLF algorithm and on its implementation, and Section 4 presents simulation results.

2. PROBLEM FORMULATION

Consider a pulsed remote sensing system operating at a fixed pulse rate. Let $\mathcal{Z} = \{(i,j) \mid i = 1,\ldots,M,\ j = 1,\ldots,N\}$ a set of sites, where j refers to the range $z_j = z_0 + j\Delta$ (z_0 and Δ are known constants) and i refers to the i-th pulse, also termed time i. Define also the lexicographical order $n = j + (i-1)N$, associated to the index set $\mathcal{Z}$

Define $y_{i,j}$, $\beta_{i,j}$, and $\alpha_{i,j}$ as, respectively, the observed signal, the backscatter coefficient, and the extinction coefficient, all at site (i,j). Define also the integrated extinction $\gamma_{i,j}$ as

$$\gamma_{i,j} \equiv \int_0^{z_j} \alpha_i(r)\, dr, \tag{2}$$

where $\alpha_i(r)$ denotes the extinction coefficient at range r and time i.

Hereinafter we use both the bidimensional indexes and the correspondent lexicographical order indifferently. For example, we can write $y_{i,j}$ or y_n. Note that the two forms are equivalent since the mapping from bidimensional to unidimensional indexes is one to one.

Observation Model

According to the rationale presented in the Introduction, the p.d.f. of the observed data given the clutter parameters has different structures, even for the same remote sensing technique. In the case of lidar, the signal y_n at the output of an incoherent receiver, has mean value and variance (see, e.g.,[9])

$$\overline{y}_n = P_n + v_d \tag{3}$$

$$\sigma^2_{y_n} = \frac{P_n^2}{m} + k(P_n + v_d) + \sigma^2_{th}, \tag{4}$$

where P_n is the mean echo power given by

$$P_n = \frac{C}{z_n^2}\, \beta_n \exp(-2\gamma_n),$$

m is the speckle count, and v_d, P_n^2/m, $k(P_n + v_d)$, and σ^2_{th} are variances of the dark current noise, of the speckle noise, of the shot noise, and of the thermal noise, respectively.

In this work we take $P_n^2/m \ll k(P_n + v_d)$, meaning that speckle noise is negligible, and assume that p.d.f. of y_n is Gaussian. This scenario accurately models large aperture lidars with high speckle count figure, where the shot noise is predominant.

Given that the mean value $\overline{y}_n$ and the variance $\sigma^2_{y_n}$ are both functions of (β_n, γ_n), via P_n, the observation model is given by

$$p(y_n|\beta_n, \gamma_n) = \mathcal{N}[\overline{y}_n(\beta_n, \gamma_n), \sigma_{y_n^2}(\beta_n, \gamma_n)], \tag{5}$$

with $\mathcal{N}(\mu, \sigma^2)$ standing for a Gaussian p.d.f. of mean μ and variance σ^2, and

$$\overline{y}_n = P_n + v_d \tag{6}$$

$$\sigma^2_{y_n} = k(P_n + v_d) + \sigma^2_{th}. \tag{7}$$

An equivalent way of presenting the observation model is writing y_n as

$$y_n = \overline{y}_n + v_n, \tag{8}$$

where v_n is an independent and identically distributed (i.i.d) zero mean Gaussian random sequence of variance $\sigma_{y_n^2}$.

Prior Model

We model the clutter parameters as a causal first order AR-GMRF,[11 12] given by

$$\beta_{i,j} = \theta_{11}\beta_{i,j-1} + \theta_{12}\beta_{i-1,j} + w_{\beta(i,j)} \quad (9)$$

$$\gamma_{i,j} = \gamma_{i,j-1} + \theta_{21}\beta_{i,j-1} + \theta_{22}\beta_{i-1,j} + w_{\gamma(i,j)}, \quad (10)$$

where $w_{\beta(i,j)}$ and $w_{\gamma(i,j)}$ are sequences of i.i.d. zero-mean Gaussian random variables of variance σ_β^2 and σ_γ^2, respectively, and $\theta_{i,j} \geq 0$, with

$$\begin{cases} \theta_{11} + \theta_{12} = 1 \\ \theta_{21} + \theta_{22} = \frac{\Delta}{B_0}. \end{cases} \quad (11)$$

In (9) and (10), when $i = 1$ and/or $j = 1$, some boundary conditions have to assumed, since indexes $i, j-1$ and/or $i-1, j$ take values outside of the index set $\mathcal{Z}$. Herein we adopt the *free boundary condition* which amounts to reduce the AR support near the boundary, in order to not include sites outside $\mathcal{Z}$.

Regression (9) gives the next backscatter coefficient $\beta_{i,j}$ as a weighted mean of the past (in a lexicographical order sense) neighbors $\beta_{i,j-1}$ and $\beta_{i-1,j}$ plus a random component. Therefore, the proposed AR prior enforces smoothness in a statistical sense, whose strength is controlled by σ_β^2.

Regression (10) accounts for the integral relation (2). In fact, if we have

$$\theta_{21}\beta_{i,j-1} + \theta_{22}\beta_{i-1,j} + w_{\gamma(i,j)} = \Delta\alpha_{i,j}, \quad (12)$$

with $\Delta = z_i - z_{i-1}$ then equation (10) would be, apart from sampling errors, a recursive implementation of the integral relation (2). Assuming a constitutive relation $\beta_n = B_0\alpha_n$ (power-law with $b = 1$), then equation (12) is satisfied for

$$\theta_{21} = \frac{\Delta}{B_0}\theta_{11} \quad (13)$$

$$\theta_{22} = \frac{\Delta}{B_0}\theta_{12} \quad (14)$$

$$w_{\gamma(i,j)} = \frac{\Delta}{B_0}w_{\beta(i,j)}. \quad (15)$$

According to (15), the random variables $w_{\gamma(i,j)}$ and $w_{\beta(i,j)}$ would be totally correlated. This is a *strong* assumption that we relax a little by allowing correlation factors $\rho \leq 1$. In this way, model mismatches can be, to some extent, absorbed by noise term $w_{\gamma(i,j)}$.

State Space Formulation

In the recursive stochastic filtering setup, the prior has necessarily a state space description. AR models (9) and (10) admit a state-space description, but its state vector would include a complete row of parameters α_n and γ_n.[12] To avoid this huge state vector, we adopt a *reduced order model* (ROM) proposed in,[10] where the state vector contains only the components whose indexes are in the support of the AR model.

The state vector is therefore $[\beta_n, \beta_{n-N}, \gamma_n, \gamma_{n-N}]^T$, and the ROM state space formulation of (9) and (10), given by

$$\underbrace{\begin{bmatrix} \beta_{n+1} \\ \beta_{n+1-N} \\ \gamma_{n+1} \\ \gamma_{n+1-N} \end{bmatrix}}_{\mathbf{x}_{n+1}} = \underbrace{\begin{bmatrix} \theta_{11} & \theta_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \theta_{21} & \theta_{22} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{A}_n} \underbrace{\begin{bmatrix} \beta_n \\ \beta_{n-N} \\ \gamma_n \\ \gamma_{n-N} \end{bmatrix}}_{\mathbf{x}_n} + \underbrace{\begin{bmatrix} 0 \\ \beta_{n+1-N} \\ 0 \\ \gamma_{n+1-N} \end{bmatrix}}_{\mathbf{u}_{n+1}} + \underbrace{\begin{bmatrix} \omega_{\beta(n)} \\ 0 \\ \omega_{\gamma(n)} \\ 0 \end{bmatrix}}_{\mathbf{w}_{n+1}}, \quad (16)$$

with the covariance matrix of $\mathbf{w}_n$ given by

$$\mathbf{Q}_n \equiv E[\mathbf{w}_n\mathbf{w}_n^T] = \begin{bmatrix} \sigma_\beta^2 & 0 & \rho\sigma_\beta\sigma_\gamma & 0 \\ 0 & 0 & 0 & 0 \\ \rho\sigma_\gamma\sigma_\beta & 0 & \sigma_\gamma^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (17)$$

As a consequence of the free boundary condition, matrix $\mathbf{A}_{i,j}$, for $\{(1,j),(i,1),\ i=1,\ldots,M\ j=1,\ldots,N\}$, takes the values

$$\mathbf{A}_{1,j} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \Delta/B_0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \qquad \mathbf{A}_{i,1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{18}$$

Notice that vector $\mathbf{u}_{n+1}$ in (16) plays the rule of a deterministic input. This is, in fact, what happens in the ROM: past state vector components not in the AR support are treated as deterministic inputs.

3. STOCHASTIC FILTERING SOLUTION

In the previous section we derived the observation equation (8) and proposed, as prior for the clutter parameters, the ROM representation (16) of the AR-GMRF (9) and (10). In summary, we have

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n + \mathbf{u}_{n+1} + \mathbf{w}_{n+1} \tag{19}$$

$$y_n = h(\mathbf{x}_n) + v_n, \tag{20}$$

where $h(\mathbf{x}_n) = \overline{y}$ given by (6) and v_n is an i.i.d. zero mean Gaussian sequence of variance $\sigma^2_{y_n}$ given by (7).

Stochastic recursive filtering is a Bayesian estimation technique that, based on the state space model (19) and (20), recursively propagates the p.d.f. $p(\mathbf{x}_n|\mathbf{Y}_n)$ (the so-called *filtering density*), where $\mathbf{Y}_n \equiv (y_n, y_{n-1}, \ldots, y_1)$. When both the state equation and the observation equation are linear, the state and observation noises are Gaussian, and the initial filtering density $p(\mathbf{x}_0)$ is Gaussian, the Kalman-Bucy linear filter yields the solution.[13] When any of these conditions is not satisfied, the problem falls into the general setup of stochastic nonlinear filtering,[14] ,[13] ,[15] ,[16] .[17]

In general, the stochastic nonlinear filters exhibit high computational complexity, what is contrast with the low complexity of the the Kalman-Bucy filter. This fact underlies the extended Kalman-Bucy filter (EKF), that is a Kalman-Bucy filter applied to a linearized version of the a nonlinear problem. The EKF although not being optimal, is frequently a good tradeoff between complexity and quality of estimates.

In the problem we are addressing, the observation equation (20) is a nonlinear function of the sate vector $\mathbf{x}_n$. Accordingly, the filtering problem is nonlinear. In the remaining part of this section we develop the nonlinear filter for the problem at hand.

3.1. Nonlinear Filter

A stochastic nonlinear recursive filter propagates, in a recursive fashion, the filtering density $p(\mathbf{x}_n|\mathbf{Y}_n)$. It worth to note that, from a Bayesian point of view, the filtering density, given the observations, carries all information about $\mathbf{x}_n$, the entity to be estimated.

The recursive propagation of the filtering density implements the following steps (for a detailed explanation see, e.g.,[16]):

1. **Prediction**

$$p(\mathbf{x}_{n+1}|\mathbf{Y}_n) = \int_{\mathbb{R}^4} p(\mathbf{x}_{n+1}|\mathbf{x}_n) p(\mathbf{x}_n|\mathbf{Y}_n)\, d\mathbf{x}_n \tag{21}$$

2. **Filtering**

$$p(\mathbf{x}_{n+1}|\mathbf{Y}_{n+1}) \propto p(y_{n+1}|\mathbf{x}_{n+1}) p(\mathbf{x}_{n+1}|\mathbf{Y}_n), \tag{22}$$

where $p(\mathbf{x}_{n+1}|\mathbf{x}_n)$ and $p(y_{n+1}|\mathbf{x}_{n+1})$ are the so-called *convolution kernel* and *observation factor*, respectively.

According to (19), and recalling that $\mathbf{w}_n$ is a zero mean Gaussian vector, the convolution kernel is given by

$$p(\mathbf{x}_{n+1}|\mathbf{x}_n) = \mathcal{N}(\mathbf{x}_{n+1} - \mathbf{A}\mathbf{x}_n - \mathbf{u}_{n+1}, \mathbf{Q}_{n+1}), \tag{23}$$

where $\mathcal{N}(\mathbf{m}, \mathbf{C})$, stands for a multidimensional Gaussian p.d.f of mean $\mathbf{m}$, and covariance matrix $\mathbf{C}$.

The prediction step given by (21) is a convolution type operation. In fact, defining $\mathbf{x}'_n = \mathbf{A}\mathbf{x}_n$ and assuming that $|A| \neq 0$, then the prediction step is written as

$$p(\mathbf{x}_{n+1}|\mathbf{Y}_n) = |\mathbf{A}|^{-1} \int_{\mathbb{R}^4} \mathcal{N}(\mathbf{x}_{n+1} - \mathbf{u}_{n+1} - \mathbf{x}'_n, \mathbf{Q}_{n+1}) p(\mathbf{A}^{-1}\mathbf{x}'_n|\mathbf{Y}_n)\, d\mathbf{x}'_n \tag{24}$$

$$= \mathcal{N}(\mathbf{x}_{n+1} - \mathbf{u}_{n+1}, \mathbf{Q}_{n+1}) * p'(\mathbf{x}'_{n+1}|\mathbf{Y}_n), \tag{25}$$

where

$$p'(\mathbf{x}|\mathbf{Y}_n) \equiv p(\mathbf{A}^{-1}\mathbf{x}|\mathbf{Y}_n). \tag{26}$$

The simplicity of Kalman-Bucy filter comes from the fact that the convolution and the product of Gaussian p.d.f.'s (prediction and filtering steps) yields Gaussian shapes.[16] Therefore, the prediction and filtering steps are given by simple linear operation over mean vectors and the covariance matrices (the first two moments).

In nonlinear filtering, at least one of the p.d.f. involved is not Gaussian, and, therefore, a second order representation can be far from optimal. Among the different techniques that have been proposed to implement, we refer to the following:

1. development in Taylor series the nonlinearities in the neighborhood of the actual estimate, and assumption of Gaussian initial condition.[13] A first order development leads to the so-called EKF
2. representation of the posterior density as elements of a finite subspace generated by Hermite polinomials,[18] Fourier series, Gaussian functions,[19] and B-splines[20]
3. sampling of the posterior density (point mass filter).[21]

Herein we adopt the point mass filter. Basically, this technique consists in computing numerically the the prediction and filtering steps (21) and (22), respectively. The point mass filter is, almost surely, the more complex implementation technique of those mentioned above. It assures, however, a neglegible error if the sampling spatial frequency high enough. On the other hand, the results provided by the point mass filter constitutes a useful benchmark to be used in comparisons with any other implementation of the NLF.

Implementation Aspects

We compute the prediction and filtering steps according to expressions (25) and (22), respectively.

The prediction is the heaviest step since, according to (25), it is given by a convolution in $\mathbb{R}^4$. We note however that the ROM model deals with the state vector components β_{n+1-N} and γ_{n+1-N} as deterministic. Therefore the convolution involves only the variables β_n and γ_n, i.e., it is a $\mathbb{R}^2$ operation.

As the filtering step given by (22) is a true convolution, its discrete version can be exactly computed by the 2D-*fast Fourier transform* algoritm over zero padded data. Since we have used rectangular grids of size 128×128, the number of floating point operations necessary to compute a convolution is $2 \times 256^2 \times \log_2(256^2)$.

The p.d.f. $p'(\mathbf{x}'|\mathbf{Y}_n)$ is obtained from $p(\mathbf{x}|\mathbf{Y}_n)$ through the coordinate transformation $\mathbf{x} = \mathbf{A_t}^{-1}\mathbf{x}'$. Given that only components β_n and γ_n matter, and referring to state equation (16), then the transformation matrix is

$$\mathbf{A_t} = \begin{bmatrix} a_{11} & 0 \\ a_{21} & 1 \end{bmatrix}. \tag{27}$$

4. SIMULATION RESULTS

The recursive stochastic NLF derived in the previous section is now applied to simulated data. Fig. 1(a) shows the assumed normalized profile of the extinction coefficient used in the simulations. It consist of two Gaussian elevations, modeling nonhomogeneous clutter, with their major axes parallel to the time coordinate. This orientation reflects quasi-stationarity of the clutter on the time dimension, a typical situation on pulsed remote sensing systems. Fig. 1(b) shows cross sections of the extinction coefficient profile at $T = 40$ and $T = 120$. We assume that the range varies between $z = 0.2$ km and $z = 5$ km and its is sampled at a spatial interval of $\Delta = 25m$.

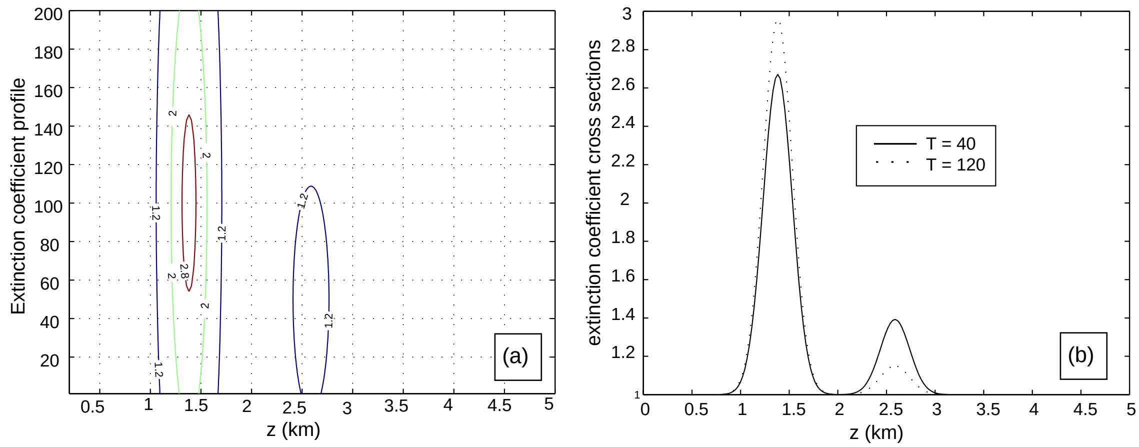

Figure 1. Normalized extinction coefficient.

Clutter Parameters			Model Parameters			System Parameters		
$\overline{\alpha}$ (km^{-1})	B_0 (sr^{-1})	b	σ_γ	σ_β	ρ	C (Wkm^3)	SNR(z_{min})	SNR(z_{max})
0.01			$0.01\,\overline{\alpha}$				50	21.2
0.1	5×10^{-2}	1.0	$0.04\,\overline{\alpha}$	$B_0/\Delta\sigma_\gamma$	0.9	8×10^5	60	24.3
0.75			$0.1\,\overline{\alpha}$				70	-16.4

Table 1. Simulation Parameters.

Table 4 summarizes the selected simulation parameters. Each value of the extintion coefficient floor $\overline{\alpha}$ models a different attenuation scenario: light attenuation ($\overline{\alpha} = 0.01$ km^{-1} $\equiv 0.043$ dB/km), moderate attenuation ($\overline{\alpha} = 0.1$ km^{-1} $\equiv 0.43$ dB/km), and severe attenuation ($\overline{\alpha} = 1$ km^{-1} $\equiv 4.3$ dB/km).

The power-law parameter $B_0 = 5 \times 10^{-2}$ is of the order of the backscatter-to-extinction ratio in optical lidar applications.

Noise variances σ_γ and σ_β are proportional to $\overline{\alpha}$, as the extinction coefficient increments in the nonhomogeneous region are also proportional to $\overline{\alpha}$.

The last two columns of Table 4 refers to the SNR at minimum and maximum ranges. This is the most important figure, in what concerns the filter performance. We have found out that a *reasonable* performance demands a SNR greater than 20dB. As in severe attenuation scenario SNR(z_{max}) $= -16.4$dB, the clutter parameters can not be recovered, at least for the complete data range.

Concerning the AR parameters, they are set in all simulations to $\theta_{11} = 0.1$, and $\theta_{12} = 0.9$. This choice is in accordance with the highly smooth nature of the clutter parameters with respect to the time coordinate: the smoothness is to imposed with more strength in the time direction.

Light Attenuation

Assume the light attenuation scenario, where ($\overline{\alpha} = 0.01$). Fig. 2, parts (a) and (c), plots cross-sections of the normalized backscattering coefficient estimate $\beta/\overline{\beta}$, respectively, at $T = 40$ and $z = 1.45$ km. Fig. 2, parts (b) and (d), plots cross-sections of the normalized extinction coefficient estimate $\alpha/\overline{\alpha}$, respectively, at $T = 40$ and $z = 1.45$ km.

The sample bias and variance of all estimates shown on Fig. 2 are below 2%. This figures are also valid for the complete images of clutter parameters.

Moderate Attenuation

We now take ($\overline{\alpha} = 0.1$). Fig. 3, parts (a) and (c), displays cross-sections of the normalized backscattering coefficient estimate $\beta/\overline{\beta}$, respectively, at $T = 40$ and $z = 1.45$ km. Fig. 3, parts (b) and (d), displays cross-sections of the normalized extinction coefficient estimate $\alpha/\overline{\alpha}$, respectively, at $T = 40$ and $z = 1.45$ km.

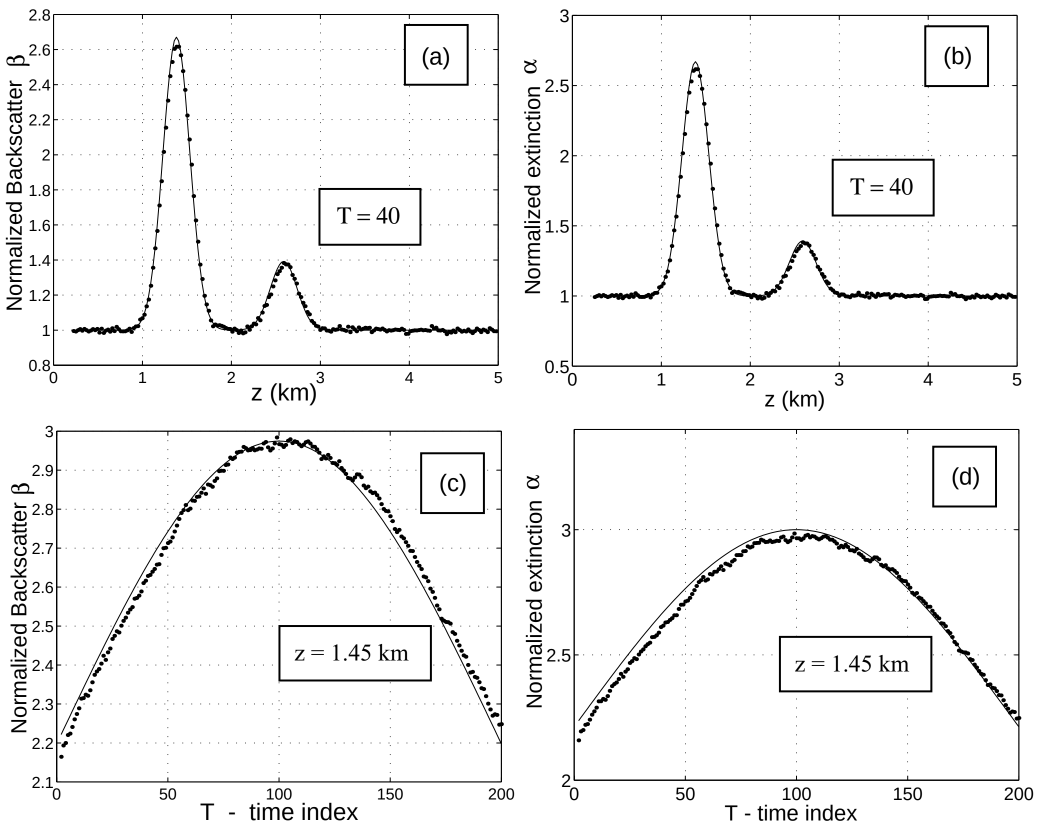

Figure 2. Cross-sections of the normalized backscattering and extinction coefficients estimates, for light attenuation ($\overline{\alpha} = 0.01$).

The sample bias and variance of estimates plotted on Fig. 3 parts (a), (b), and (c) are below 1%, whereas the sample bias and variance of extinction coefficient plotted on part (d) are below 1% and 4, respectively. This figures is also valid for the complete images of clutter parameters.

Severe Attenuation

We now take ($\overline{\alpha} = 0.75$). Fig. 4, parts (a) and (c), shows cross-sections of the normalized backscattering coefficient estimate $\beta/\overline{\beta}$, respectively, at $T = 40$ and $z = 1.45\,\text{km}$. Fig. 4, parts (b) and (d), shows cross-sections of the normalized extinction coefficient estimate $\alpha/\overline{\alpha}$, respectively, at $T = 40$ and $z = 1.45\,\text{km}$.

The results ploted in Fig. 4 exhibits a behaviour that we have systematically found: for the clutter parameter estimates to be meaningful, the SNR must be greater than, roughly, 20dB. In the severe attenuation scenario a SNR of 20dB is reached at $z = 2.2$km (see Fig. 5). Notice from Fig. Fig.s 4 parts (a) and (b), that the estimated clutter parameters are useless for $z \gtrsim 2.2$km.

For $z \lesssim 2.2$km, the sample bias of estimates plotted on Fig. 3 parts (a), (b), and (c) and (d) is below 2%, whereas the sample variance of the same estimates is below 4%. This figures are also valid for the image region where SNR$\geq$20dB.

The slight degradation of the filter performance, compared with the light and moderate situations, is due to a higher value of state equation noise variance. We have used $0.1a_0$ against $0.03a0$ and $0.01a0$ in the previous simulations. The reason underlying this choice is that, despite it leads to a higher variance of the clutter estimates, it increases the range of meaningful estimates.

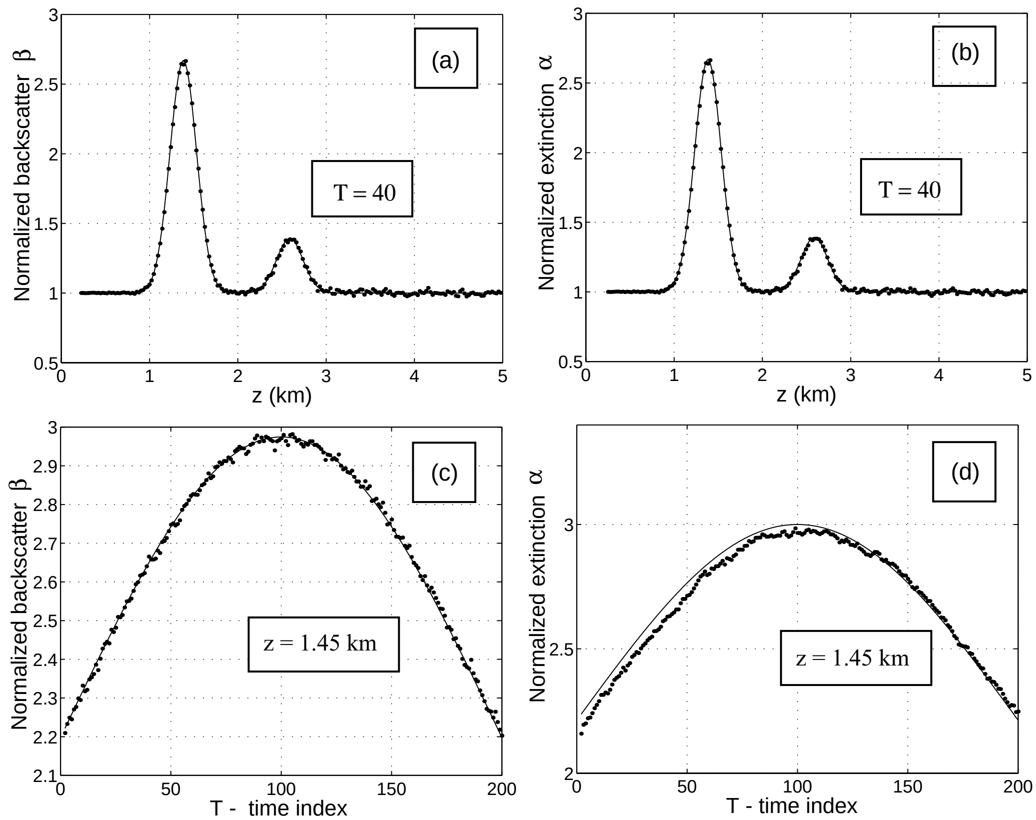

Figure 3. As in Fig. 2, for moderate attenuation ($\overline{\alpha} = 0.1$).

Robustness and Comparison with the Klets' Method

The NLF is very robust with respect to its parameters, which are σ_α, σ_β, ρ, θ_{11}, and θ_{12}. A change of 10 in any of this parameters do not produce an appreciable change in the correspondent estimates.

Fig. 6 shows the backscatering coefficient estimates generated by the NLF and the Klett's method, for $\overline{\alpha} = 0.25\,\mathrm{km}^{-1}$ and $T = 40$. As it would be expected the stochastic approach produces much better results.

5. CONCLUDING REMARKS

A new stochastic nonlinear filtering technique for radar/lidar inversion of the backscatering and extinction coefficients was developed. A Bayesian approach was followed. The prior model adopted for the original image is a first-order auto regressive Gauss-Markov random field. A reduced order state-space representation of the prior allowed to tackle the estimation of the clutter parameters under the stochastic nonlinear filtering framework. The recursive filter was implemented using a two dimensional point mass filter. The prediction step of each recursion, the more complex from the computational point of view, was implemented efficiently, via a two-dimensional fast Fourier transform.

The nonlinear filter was applied to Gaussian data with the same mean and variance, typical of devices operating at *high* intensity shot noise. The filter yields very good results for signal to noise ratios greater than 20dB. For signal to noise ratios below this threshold, the results become useless. Concerning robustness the proposed technique exhibits good properties with respect to the prior parameters: a change of 10% on this parameters did not produce an noticiable change on the correspondent estimates.

Regarding future work, we foresee two directions:

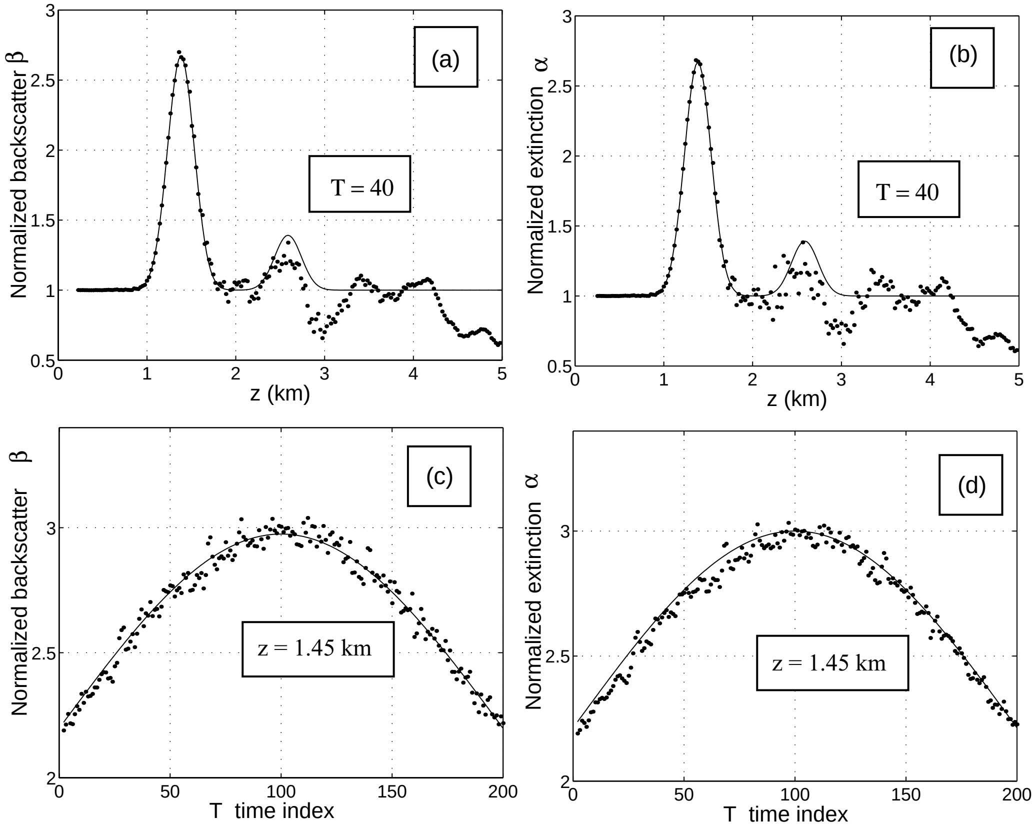

Figure 4. As in Fig. 2, for severe attenuation ($\overline{\alpha} = 0.75$).

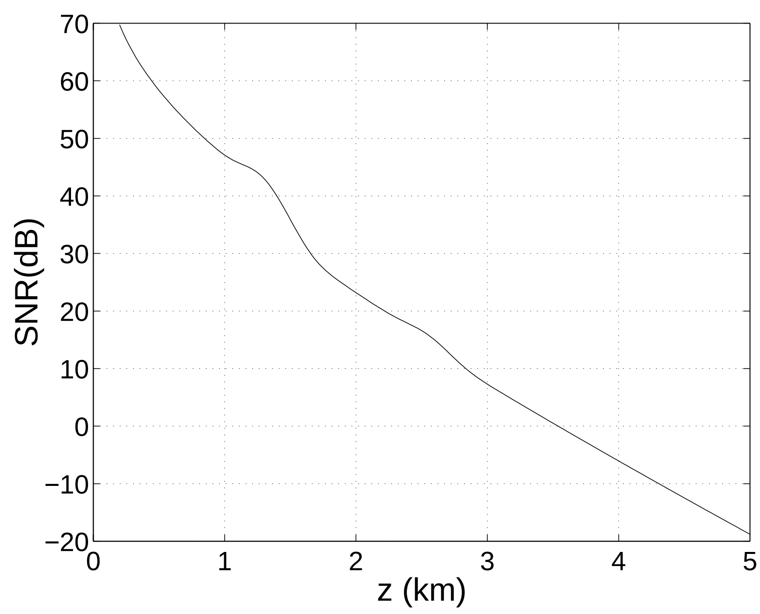

Figure 5. Signal to noise ratio at $T = 40$ and ($\overline{\alpha} = 0.75$).

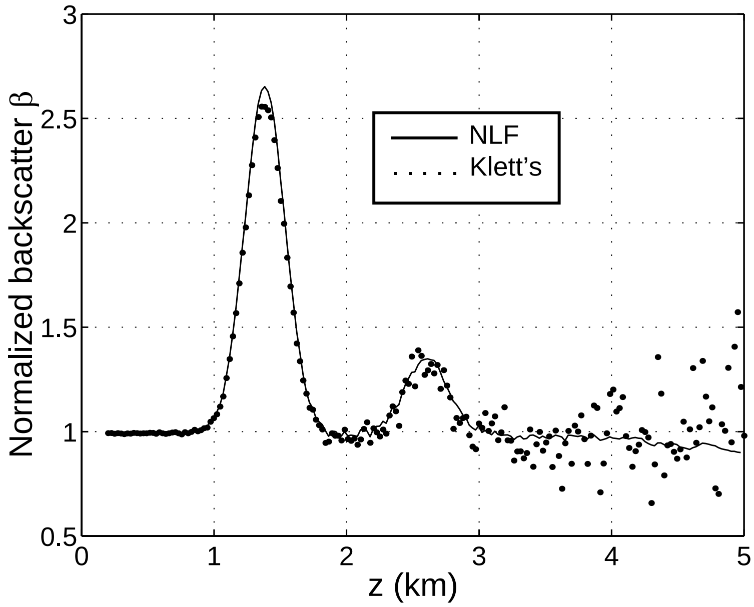

Figure 6. Comparison with the Klett's method for $\overline{\alpha} = 0.25\,\text{km}^{-1}$. Solid lines denotes the NLF estimates whereas dots denotes the Klett's method estimates.

1. extensive comparison with the extended Kalman-Bucy filter to find out where it pays to use the nonlinear solution

2. efficient implementation of the nonlinear filter, this involving a suitable representation of observation factor.

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