Analysis of Surface and Semileaky Modes in Chiral Optical Fibers Using the Condon Model to Account for Dispersion in Optically Active Media

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Abstract

Using the Condon model to describe the dispersive behavior of optically active media, we analyze two different classes of waves in chiral optical fibers: surface and semileaky modes. Only when the cladding is chiral, semileaky modes do exist; accordingly, optical fibers with either chiral or achiral cores, but always with a chiral cladding, are considered. Two fundamental modes, corresponding to the two different classes of waves, were found. However, only when both core and cladding are chiral, mode coupling was also found.

1. Introduction

In applied electromagnetics the theory of guided wave propagation has always played an important role. Therefore, when the study of complex media has arisen in the research of electromagnetists, an extensive discussion of waveguides filled with these new kinds of materials also began to flourish in the literature. In the specific case of chirowaveguides, a good account of the corresponding research can be found in [1].

In planar dielectric chirowaveguides semileaky modes can propagate [2]. More recently, it was also shown that semileaky modes can occur in chiral optical fibers [3]. In [4] the Condon model [5] was found to be a good approximation to study the material and waveguide dispersion in a planar chirowaveguide. An extensive discussion of the dispersive behavior of both surface and semileaky modes in dielectric planar chirowaveguides was developed in [6]. In this paper we extend the analysis presented in [6] to chiral optical fibers.

In common isotropic (i.e., achiral) optical fibers only surface modes can propagate – provided that the contribution of the continuous spectrum of radiation and evanescent modes is discarded. In chiral optical fibers both surface and semileaky modes can exist: with a chiral cladding but with either chiral or achiral cores, the leakage may occur if one of the core's two characteristic waves ceases to be totally internally reflected at the core-cladding interface while the other wave still undergoes total internal reflection. Using the Condon model to account for dispersion in optically active media, we show in this paper that, in optical fibers with a chiral cladding, there is a fundamental semileaky mode as well as a fundamental surface mode – with either chiral or achiral cores.

2. Chiral Optical Fibers

The constitutive relations used to describe chiral media are

$$\begin{cases} \mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E} + i\sqrt{\varepsilon_0 \mu_0}\,\chi \mathbf{H} = \varepsilon_0 \varepsilon \mathbf{E} + ig\omega \mathbf{H} \\ \mathbf{B} = \mu_0 \mu \mathbf{H} - i\sqrt{\varepsilon_0 \mu_0}\,\chi \mathbf{E} = \mu_0 \mu \mathbf{H} - ig\omega \mathbf{E} \end{cases} \tag{1}$$

where ε is the relative permittivity, μ is the relative permeability and χ is the chirality parameter. To describe the dispersive behavior of chiral media, the Condon model [5] is used, where the gyrotropic parameter g was introduced. In this model ε and μ are constants and χ varies linearly with frequency ω according to $\chi(\omega) = \omega g / \sqrt{\varepsilon_0 \mu_0} = k_0 c^2 g$. Optical activity vanishes in the steady-state regime. This is a good approximation to the single resonance model developed in [4]. The cylindrical structure in study is an optical fiber where both core and cladding may be chiral. The core has radius $r = a$ and is characterized by parameters $(\varepsilon_1, \mu_1, \chi_1)$, whereas the cladding is unlimited and characterized by $(\varepsilon_2, \mu_2, \chi_2)$.

Using Bohren's decomposition and applying the boundary conditions at the interface $r = a$, the modal equation $M_{11} M_{22} - M_{12} M_{21} = 0$ is obtained, with

$$M_{11} = m(\beta a)\left[\frac{1}{u_+^2} + \frac{(1+\eta)}{2w_+^2} + \frac{(1-\eta)}{2w_-^2}\right]\rho_+ + (\beta_+ a)\sigma_+ + \left[(1+\eta)(\gamma_+ a)\tau_+ - (1-\eta)(\gamma_- a)\tau_-\right]\rho_+ \tag{2a}$$

$$M_{12} = m(\beta a)\left[\frac{1}{u_-^2} + \frac{(1-\eta)}{2w_+^2} + \frac{(1+\eta)}{2w_-^2}\right]\rho_- - (\beta_- a)\sigma_- + \left[(1-\eta)(\gamma_+ a)\tau_+ - (1+\eta)(\gamma_- a)\tau_-\right]\rho_- \tag{2b}$$

$$M_{21} = m(\beta a)\left[\frac{\eta}{u_+^2} + \frac{(1+\eta)}{2w_+^2} - \frac{(1-\eta)}{2w_-^2}\right]\rho_+ + \eta(\beta_+ a)\sigma_+ + \left[(1+\eta)(\gamma_+ a)\tau_+ + (1-\eta)(\gamma_- a)\tau_-\right]\rho_+ \tag{2c}$$

$$M_{22} = m(\beta a)\left[-\frac{\eta}{u_+^2} + \frac{(1-\eta)}{2w_+^2} - \frac{(1+\eta)}{2w_-^2}\right]\rho_- + \eta(\beta_- a)\sigma_- + \left[(1-\eta)(\gamma_+ a)\tau_+ + (1+\eta)(\gamma_- a)\tau_-\right]\rho_- \tag{2d}$$

and where $\eta = y_2 / y_1$. Constants $u_\pm = h_\pm a$ are the normalized transverse wave numbers in the core and $w_\pm = \alpha_\pm a$ the normalized attenuation constants in the cladding, with $h_\pm^2 = \beta_\pm^2 - \beta^2$ and $\alpha_\pm^2 = \beta^2 - \gamma_\pm^2$. Constants $\beta_\pm = p_\pm k_0 = \left(\sqrt{\varepsilon_1 \mu_1} \pm \chi_1\right)$ and $\gamma_\pm = q_\pm k_0 = \left(\sqrt{\varepsilon_2 \mu_2} \pm \chi_2\right)$ are propagation constants for the eigenwaves in unbounded chiral media. The following auxiliary parameters were also introduced: $\rho_\pm = \mathrm{J_m}(u_\pm)$, $\sigma_\pm = \mathrm{J'_m}(u_\pm)/u_\pm$, $\tau_\pm = \mathrm{K'_m}(w_\pm)/\left[2w_\pm \mathrm{K_m}(w_\pm)\right]$.

In general the longitudinal wave number takes the form $\beta = n_{eff} k_0 + i(\alpha/2)$ [3], where n_{eff} is the effective refractive index and α is the power loss coefficient due to leakage loss (in semileaky modes). As a result, the numerical search must be performed in the complex plane of β.

3. Numerical Results

The dispersion diagrams obtained by solving the modal equation are presented as a function of the normalized frequency $v = k_0 a \sqrt{\varepsilon_1 \mu_1 - \varepsilon_2 \mu_2}$ for a constant value of the normalized gyrotropic parameter $\bar{g} = \chi / v$. All results presented were obtained considering non-magnetic materials $(\mu_1 = \mu_2 = 1)$ with $n_1 = \sqrt{\varepsilon_1 \mu_1} = 2$ and $n_2 = \sqrt{\varepsilon_2 \mu_2} = 1.5$. Two cases are considered: (i) only the cladding is chiral; (ii) both core and cladding are chiral. The mode labeling convention is based on the dominant character of the corresponding mode at cutoff, i.e., whether an R (L) mode is dominantly of the RCP (LCP) type at cutoff.

3.1 Achiral Core

In Fig. 1 the dispersion diagrams for modes with azimuthal parameter $m = \pm 1$, in a chiral fiber where the cladding has $\bar{g}_2 = 0.04$, are presented. At cutoff, L modes are semileaky modes whereas R modes are surface modes. While some L modes may become completely guided (e.g., L_{-10}), all modes eventually become semileaky as surface modes can only exist as long as $n_1 > q_+$. Mode R_{10} is the fundamental surface mode whereas L_{-10} is the fundamental semileaky mode.

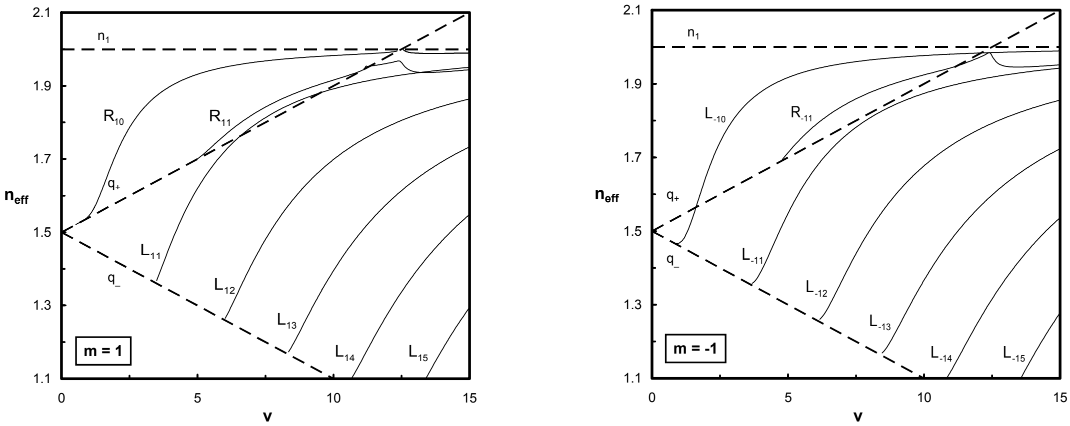

Fig. 1 – Dispersion diagram for modes with $m = \pm 1$ and $\bar{g}_2 = 0.04$ (achiral core).

In a semileaky mode the RCP component is unguided and hence responsible for leakage; the LCP component remains guided. In an R type semileaky mode, it is the dominant component that is radiating with a very high leakage loss. By the contrary, in an L type semileaky mode, leakage losses are smaller as the dominant component remains guided, as shown in Fig. 2.

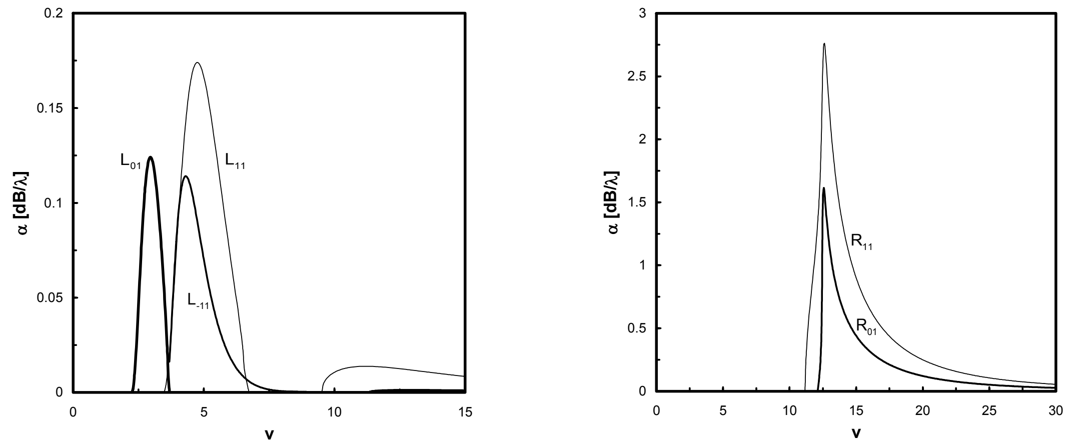

Fig. 2 – Leakage loss for semileaky modes (achiral core).

3.2 Chiral Core

When both core and cladding are chiral, with $\bar{g} = \bar{g}_1 = \bar{g}_2$, the dispersive behavior differs substantially from the previous case. In Fig. 3 the dispersion diagrams for $m = \pm 1$ in a chiral fiber with $\bar{g} = 0.02$ are shown. Hybrid modes with an RCP dominant component approach p_+ as frequency increases, whereas hybrid modes with a dominant LCP component tend asymptotically to p_-. However, there are L modes that approach p_+ and R modes that approach p_-. For example, L_{-10} is a semileaky mode at cutoff and becomes a surface mode at $v \approx 1.4$. This mode has a tendency to approach p_-, until it couples with mode R_{-11}, at $v \approx 5.3$, while interchanging their corresponding characteristics. For $v > 5.3$, mode L_{-10} is dominantly RCP and is asymptotically approaching p_+. There are also two fundamental modes: one is a semileaky mode $\left(L_{-10}\right)$ while the other is a surface mode $\left(R_{10}\right)$.

Fig. 4 shows the leakage loss for a few semileaky modes. Since, in this case, all semileaky modes are dominantly of the LCP type, there are no high leakage losses as those encountered in the R semileaky modes of Fig. 2. In modes approaching p_-, leakage loss become periodically null. This occurs because the component responsible for the leakage (RCP) vanishes at those frequencies.

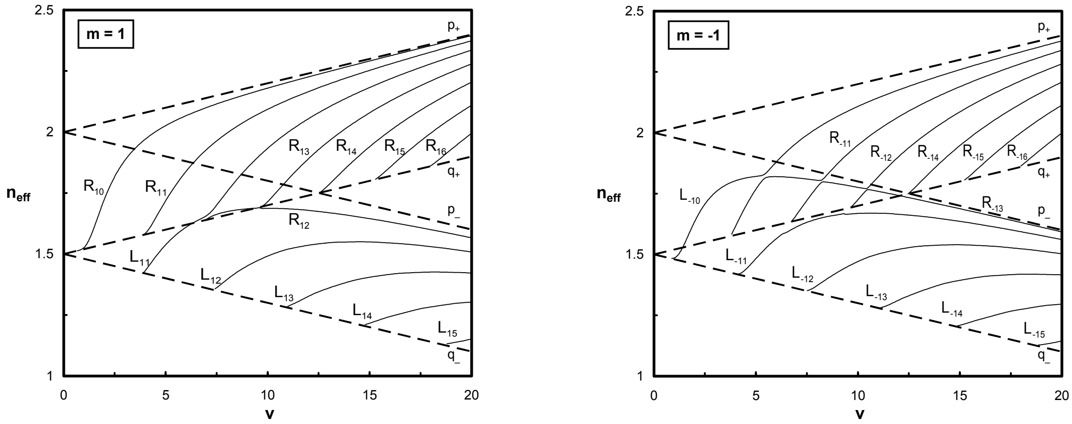

Fig. 3 – Dispersion diagram for modes with $m = \pm 1$ and $\overline{g} = 0.02$ (chiral core).

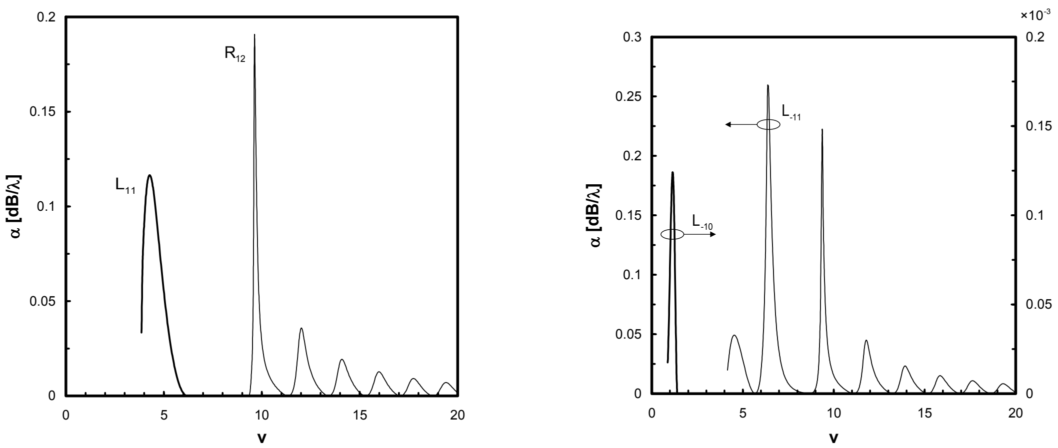

Fig. 4 – Leakage loss for semileaky modes (chiral core).

4. Conclusion

By using the Condon model for optically active media, it is possible to take into account material dispersion in the analysis of the overall dispersion in chirowaveguides. In this paper, the modal equation for chiral optical fibers was numerically solved to obtain (i) the dispersion diagrams for surface and semileaky modes as well as (ii) the leakage loss for semileaky modes. Two cases were considered: achiral and chiral cores (always with a chiral cladding). In both cases, two fundamental modes were found for the two types of modes under study. Mode coupling was also found – however, only for the special case when core and cladding are both chiral.

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