

2012 SIAM Conference on Applied Linear Algebra  
Minisimposium Application of compressed sensing in bio-medicine  
Valencia, Spain, June 18<sup>th</sup> 2012

# Evaluation of Compressed Sensing Impact in Cardiac Signals Processing and Transmission

Eduardo Pinheiro  
Octavian Postolache  
Pedro Girão

[eduardo.pinheiro@lx.it.pt](mailto:eduardo.pinheiro@lx.it.pt)

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e Tecnologia da  
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TwIST

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SPARSA

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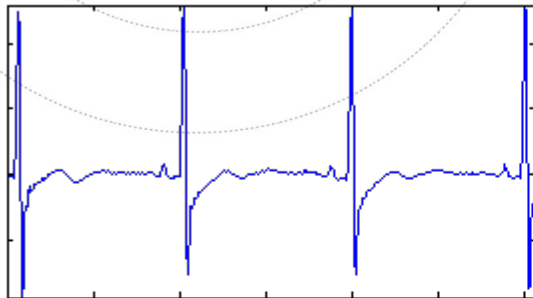


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# Motivation – Cardiac Signals

Electro, myocardium electrical stimulus

ECG



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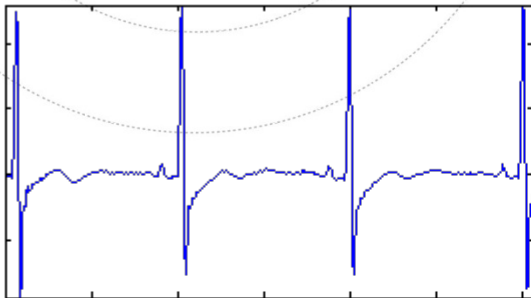
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# Motivation – Cardiac Signals

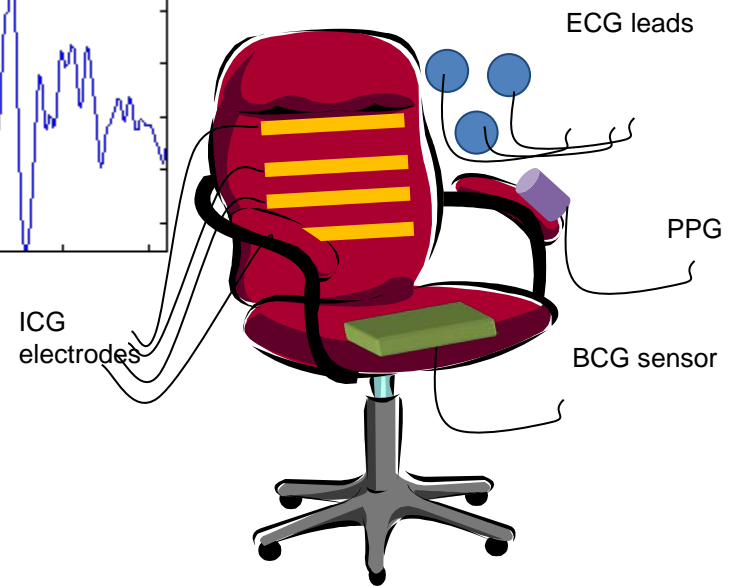
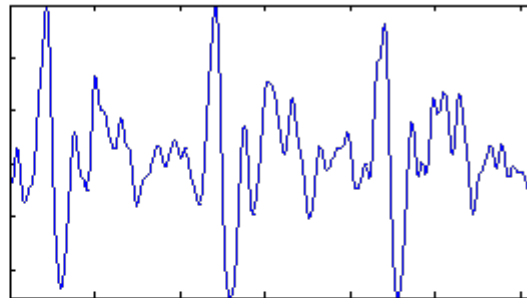
Electro, myocardium electrical stimulus

Ballisto, force variations during cardiac cycle

ECG



BCG



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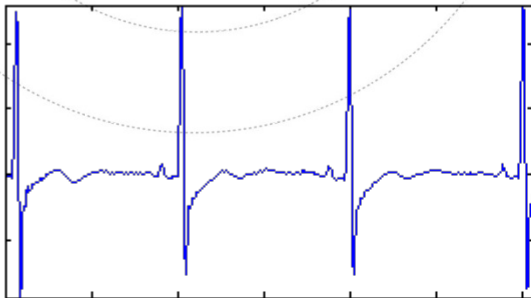
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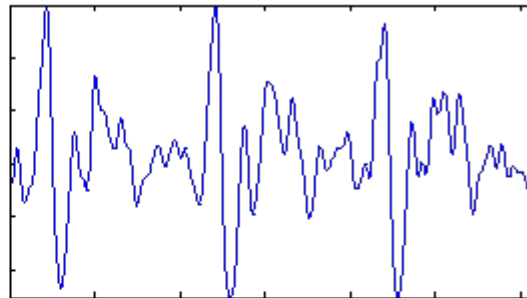
Ballisto, force variations during cardiac cycle

Photoplethys, tissues' transmittance of IR/R light due to blood absorption

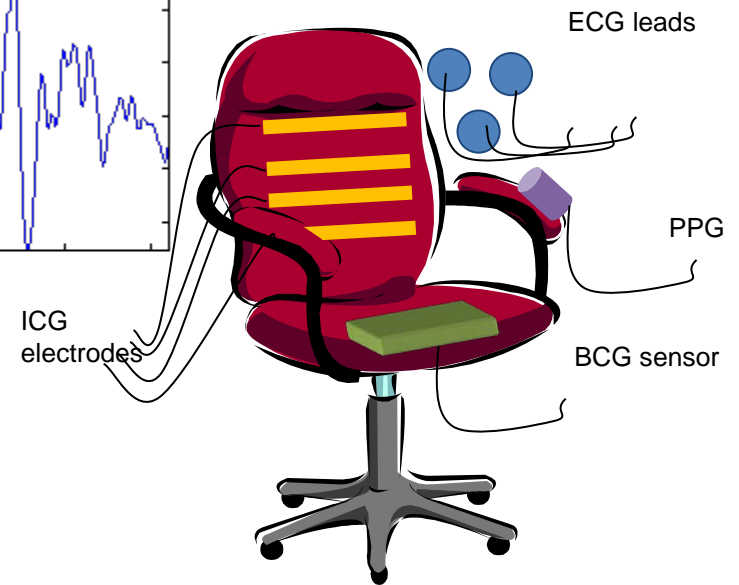
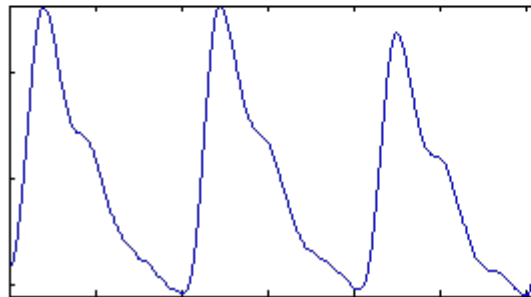
ECG



BCG



PPG



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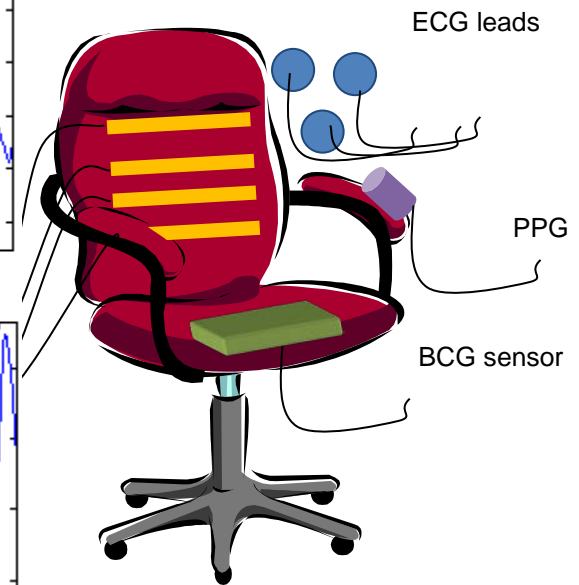
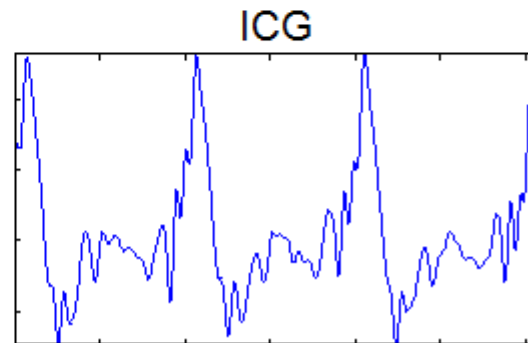
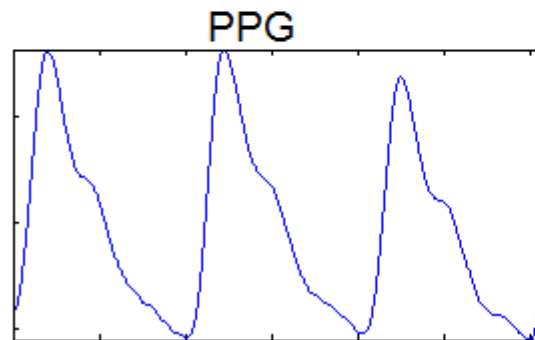
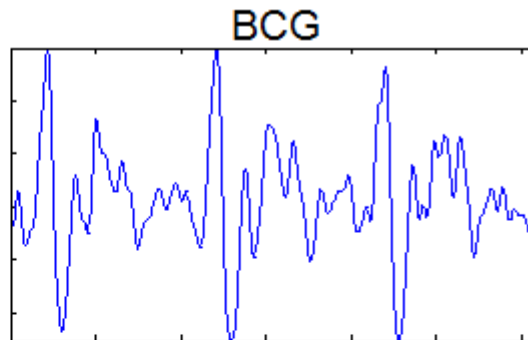
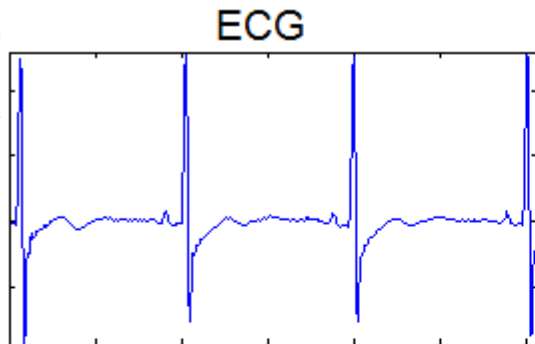
# Motivation – Cardiac Signals

Electro, myocardium electrical stimulus

Ballisto, force variations during cardiac cycle

Photoplethys, tissues' transmittance of IR/R light due to blood absorption

Impedance, impedance changes due to blood passage



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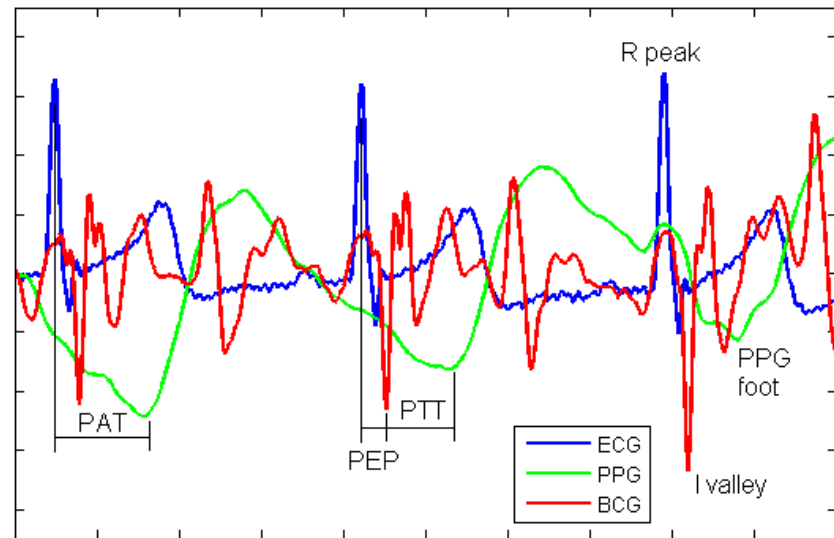
# Motivation – Which relations?

ECG sync

PPG max derivative => systolic blood pressure

BCG valley => cardiac output, stroke volume

ICG => left ventricular ejection time



# Motivation – Which tradeoffs?

Information

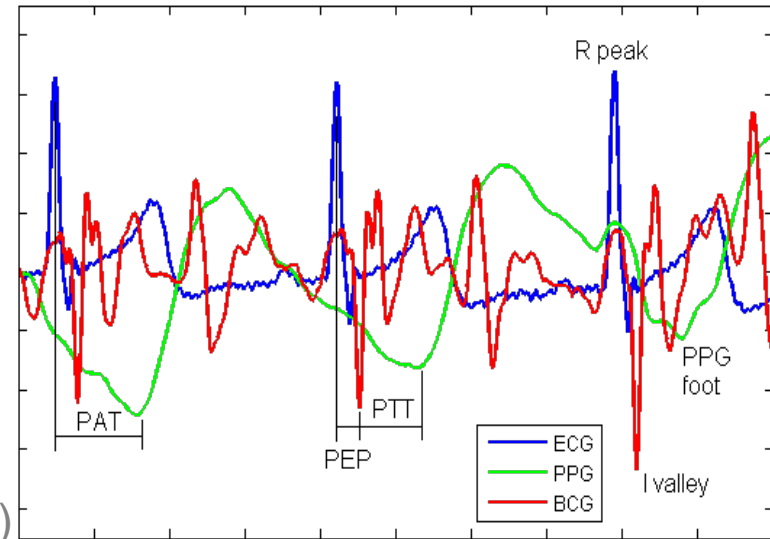
Blood Pressure, HRV, arrest?

Problems

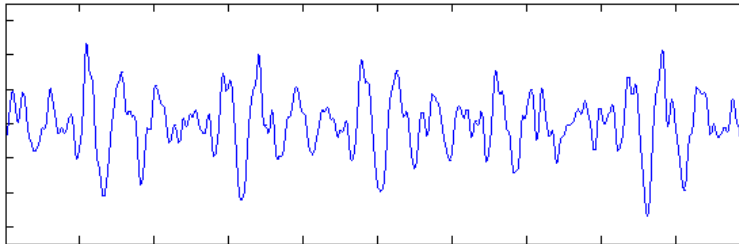
Artifact sensibility (motion, talk, breath)

Morphology dependent on measurement conditions

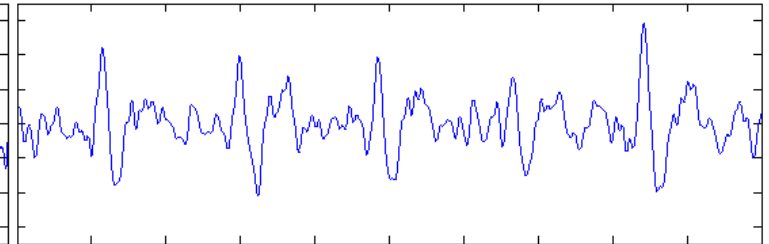
## Medical staff demands exact signals



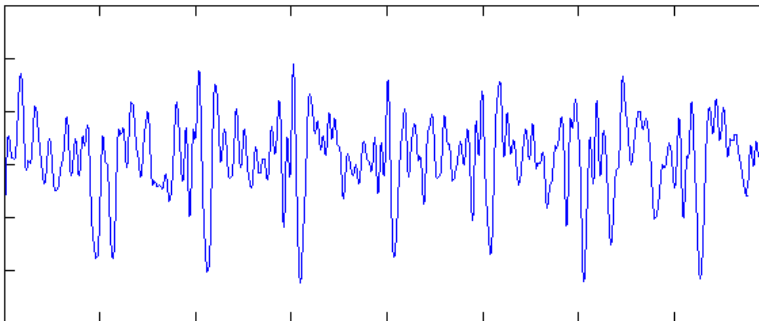
Backrest BCG



Seat BCG



Stented subject



Teenager and subject with coronary stent

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# Approaches on CS

A time signal  $x$ , a vector composed of  $N$  samples, is  $K$ -sparse or compressible in a basis  $\Psi$ , represented by an  $N \times N$  matrix in which each column is a basis vector  $\psi_i$ , if  $x$  can be well approximated by a linear combination of only  $K$  vectors of  $\Psi$ , with  $K$  significantly  $< N$ . Most  $(N-K)$  of the expansion coeffs  $\alpha$ , in the representation  $x = \Psi\alpha$  are negligible if compared to the dominant terms.

The signal can be reconstructed from a number of  $M$ , slightly  $>K$  but still  $\ll N$ , linear projections of  $x$  onto another basis  $\Phi$  incoherent with  $\Psi$ , where incoherency signifies that the elements of  $\Phi$  cannot represent sparsely the elements of  $\Psi$  and vice-versa,  $y = \Phi x = \Phi\Psi\alpha$

$x = \Psi\alpha$  has  $K$  non-zero  $\alpha_i \Rightarrow x$  (of size  $N \gg K$ ) is  $K$ -sparse in  $\Psi$

$$\hat{\alpha} = \arg \min_{\alpha} \|\alpha\|_{l_1} \text{ subject to } \Phi\Psi\alpha = y$$

$y = \Phi x = \Phi\Psi\alpha$  ( $M > K$  projections of  $x$  onto  $\Phi$ )  $\Rightarrow x$  is described by  $y$

$$\hat{\alpha} = \arg \min_{\alpha} \|y - \Phi\Psi\alpha\|_{l_2}^2 + \tau \|\alpha\|_{l_1}$$

$$\underline{M \ll N}$$

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# Approaches on CS

## Two-step Iterative Shrinkage/Thresholding

Two-step iterative shrinkage/thresholding algorithm overcoming the dependence on the linear observation operator by implementing a nonlinear two-step (or 2<sup>nd</sup> order) version of IST.

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# Approaches on CS

## Two-step Iterative Shrinkage/Thresholding

Two-step iterative shrinkage/thresholding algorithm overcoming the dependence on the linear observation operator by implementing a nonlinear two-step (or 2<sup>nd</sup> order) version of IST.

## Split Augmented Lagrangian Shrinkage Algorithm

Adapt of Bregman iterations. Constrained problem set to an unconstrained by adding the indicator function of the feasible set, the ellipsoid  $\{x : \|Bx-y\| \leq \varepsilon\}$ . Then a different constrained problem, by applying a variable splitting operation; finally the problem is attacked with an augmented Lagrangian (AL) scheme, a variant of the ADMM

# Approaches on CS

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## Sparse Reconstruction by Separable Approximation

2011 IEEE SPS Best Paper Award. Optimization subproblem involving a quadratic term with diagonal Hessian plus the original sparsity-inducing regularizer; suitable for cases in which this subproblem can be solved much more rapidly than the original problem

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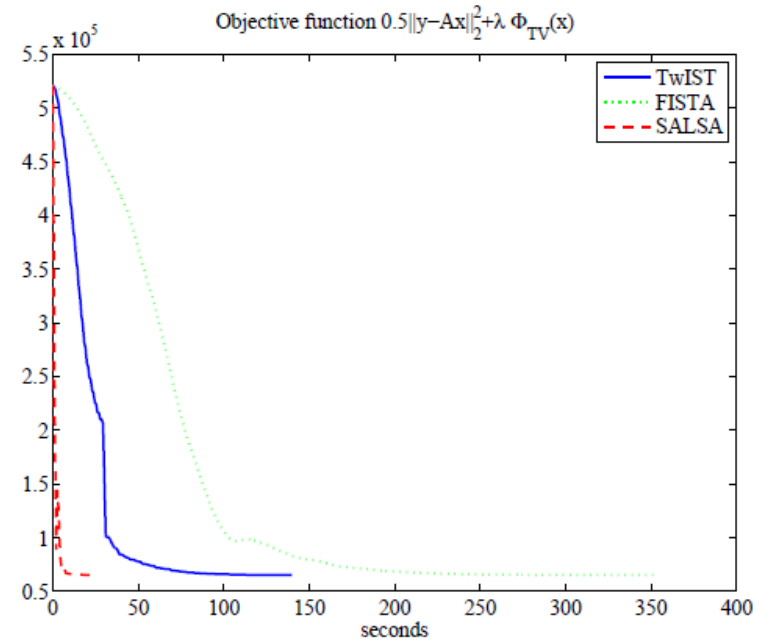
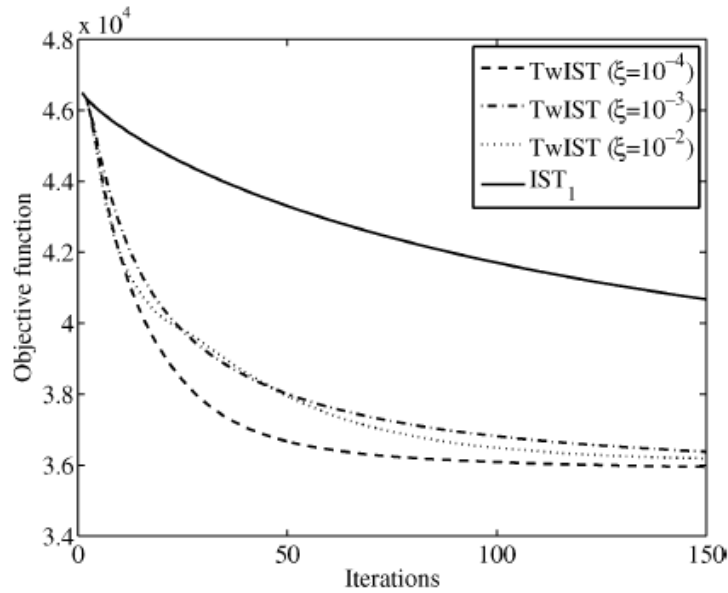


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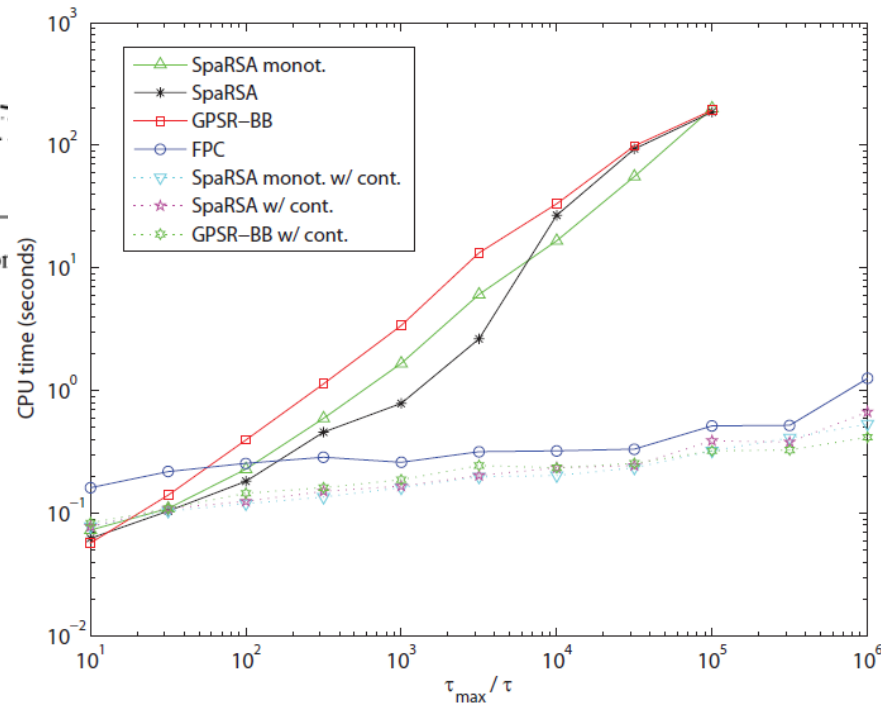
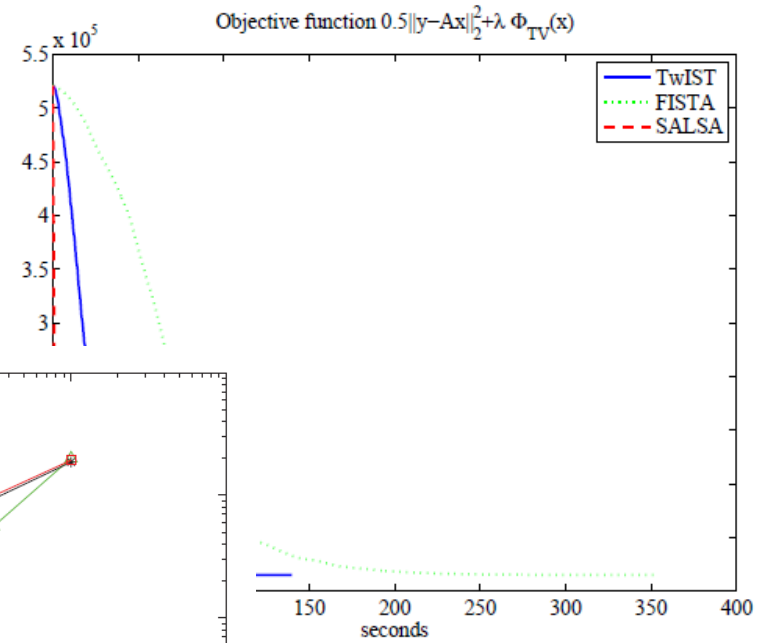
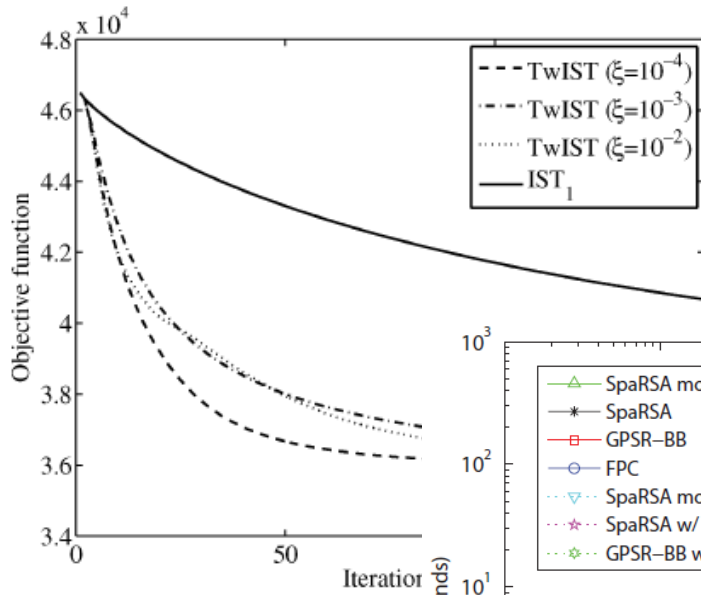
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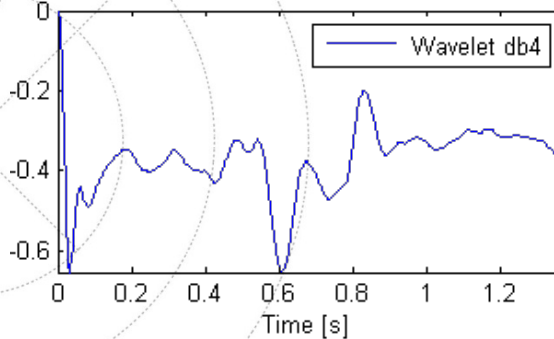
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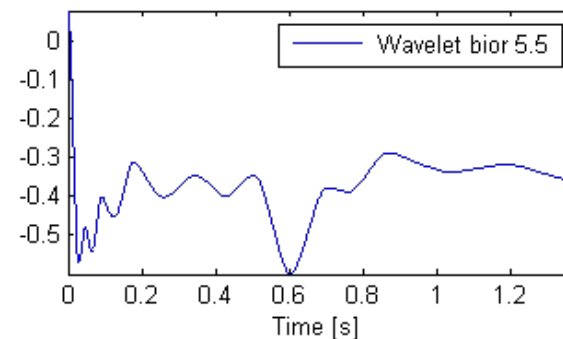
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# Cardiac Signals Processing

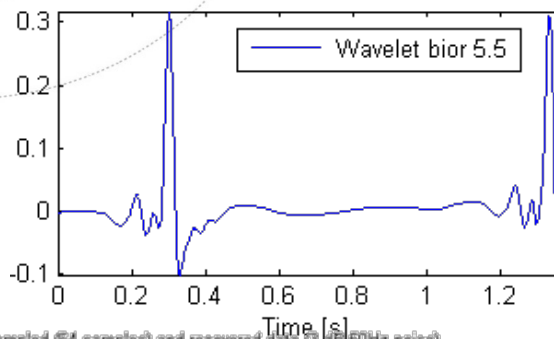
BCG reconstruction with 0.39% nRMSD



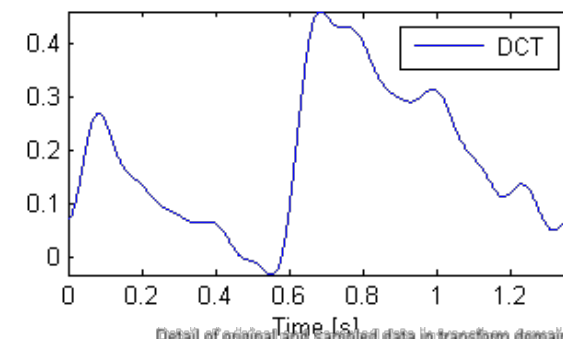
BCG reconstruction with 6.52% nRMSD



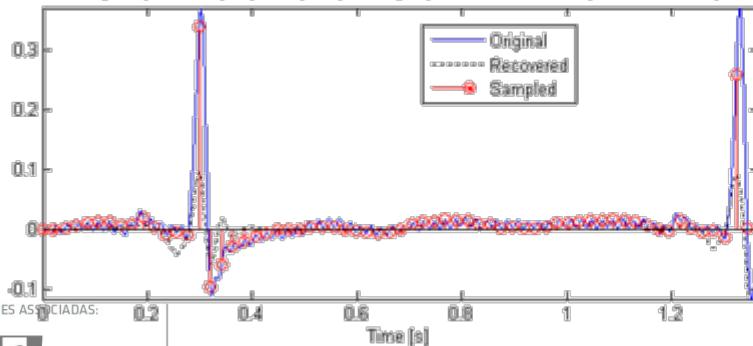
ECG reconstruction with 2.51% nRMSD



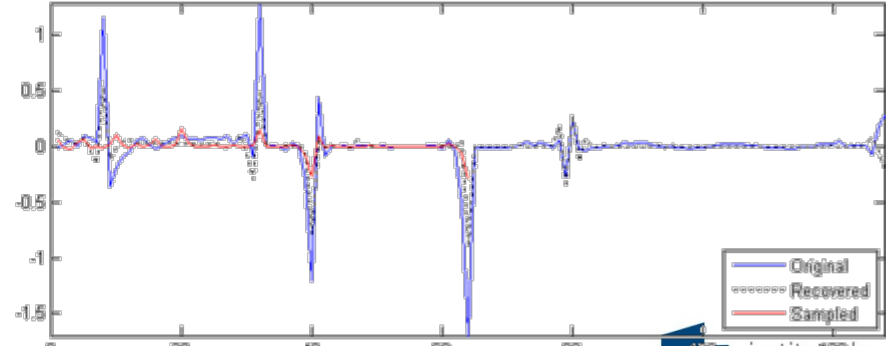
PPG reconstruction with 4.29% nRMSD



Original (2048 samples), sampled (64 samples) and recovered data (9 dB/50Hz noise)



Detail of original and sampled data in transform domain



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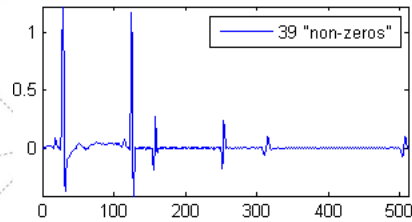
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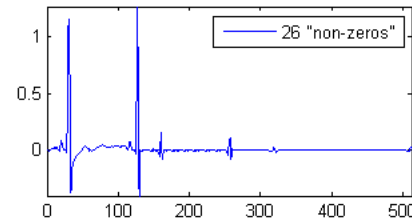
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# ECG - K

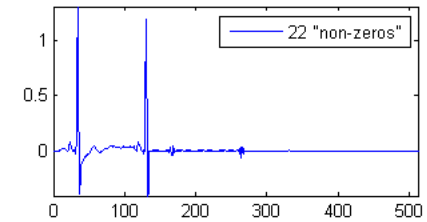
Wavelet haar transform with detail 4



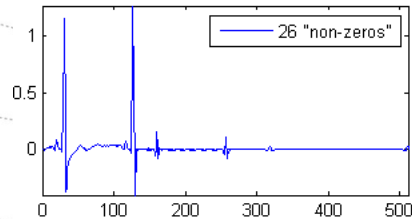
Wavelet db2 transform with detail 4



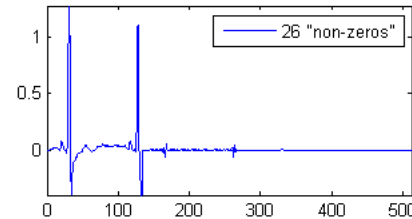
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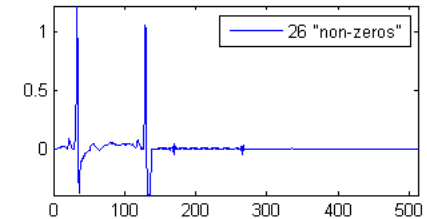
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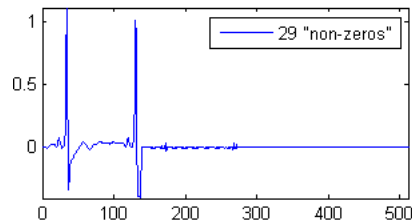
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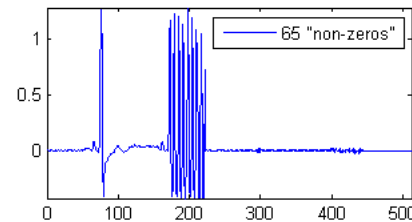
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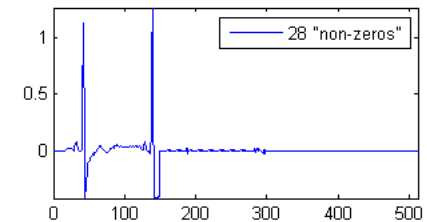
Wavelet bior 5.5 transform with detail 4



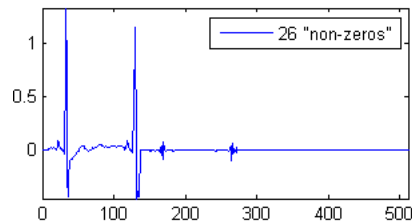
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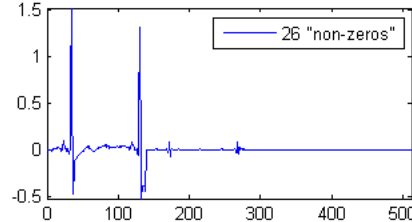
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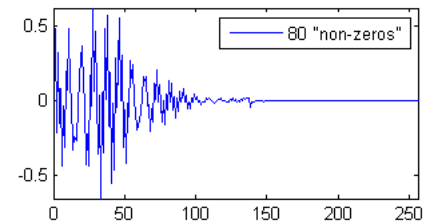
Wavelet rbio 4.4 transform with detail 4



Wavelet rbio 5.5 transform with detail 4



DCT



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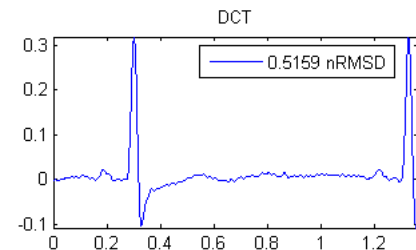
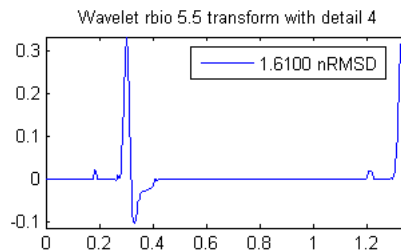
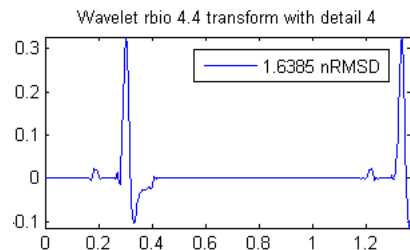
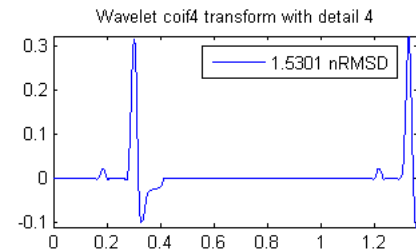
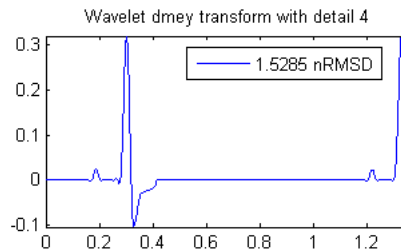
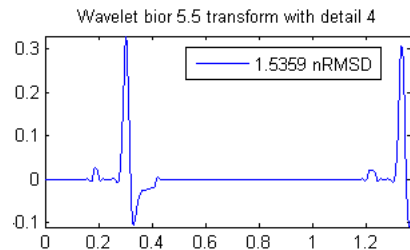
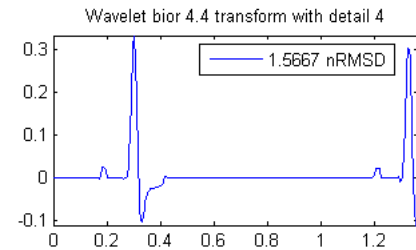
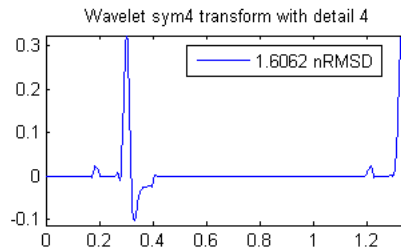
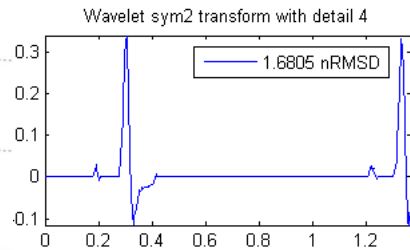
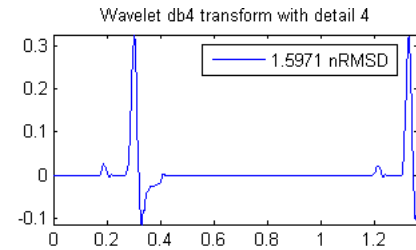
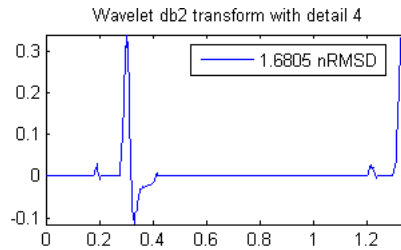
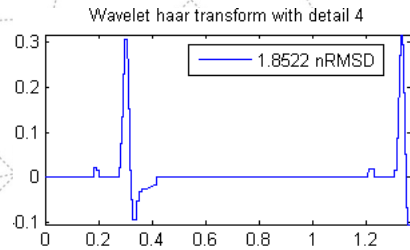
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# ECG – rec



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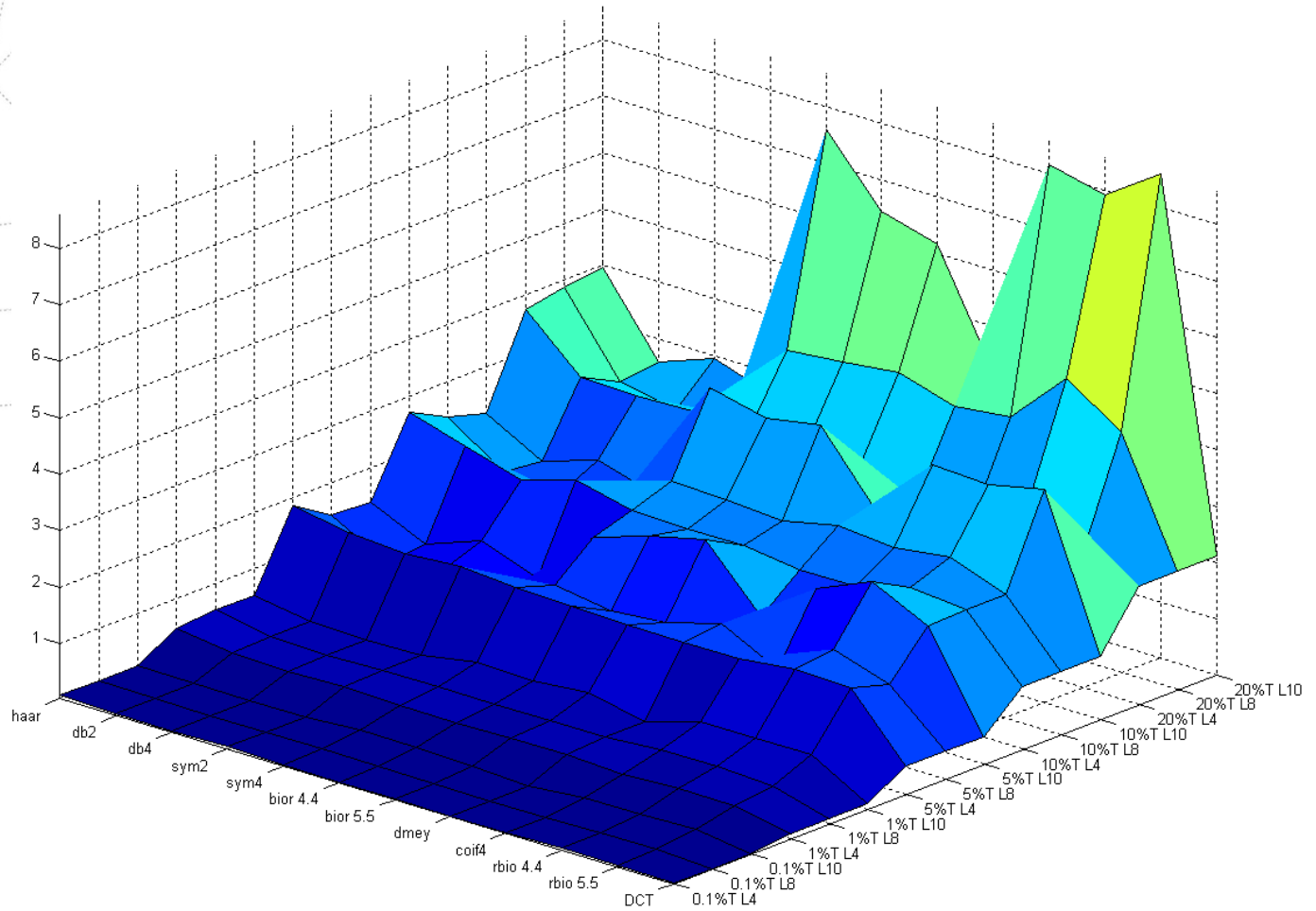
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# ECG – nRMSD

NRMSD [%] for different transformations and truncations



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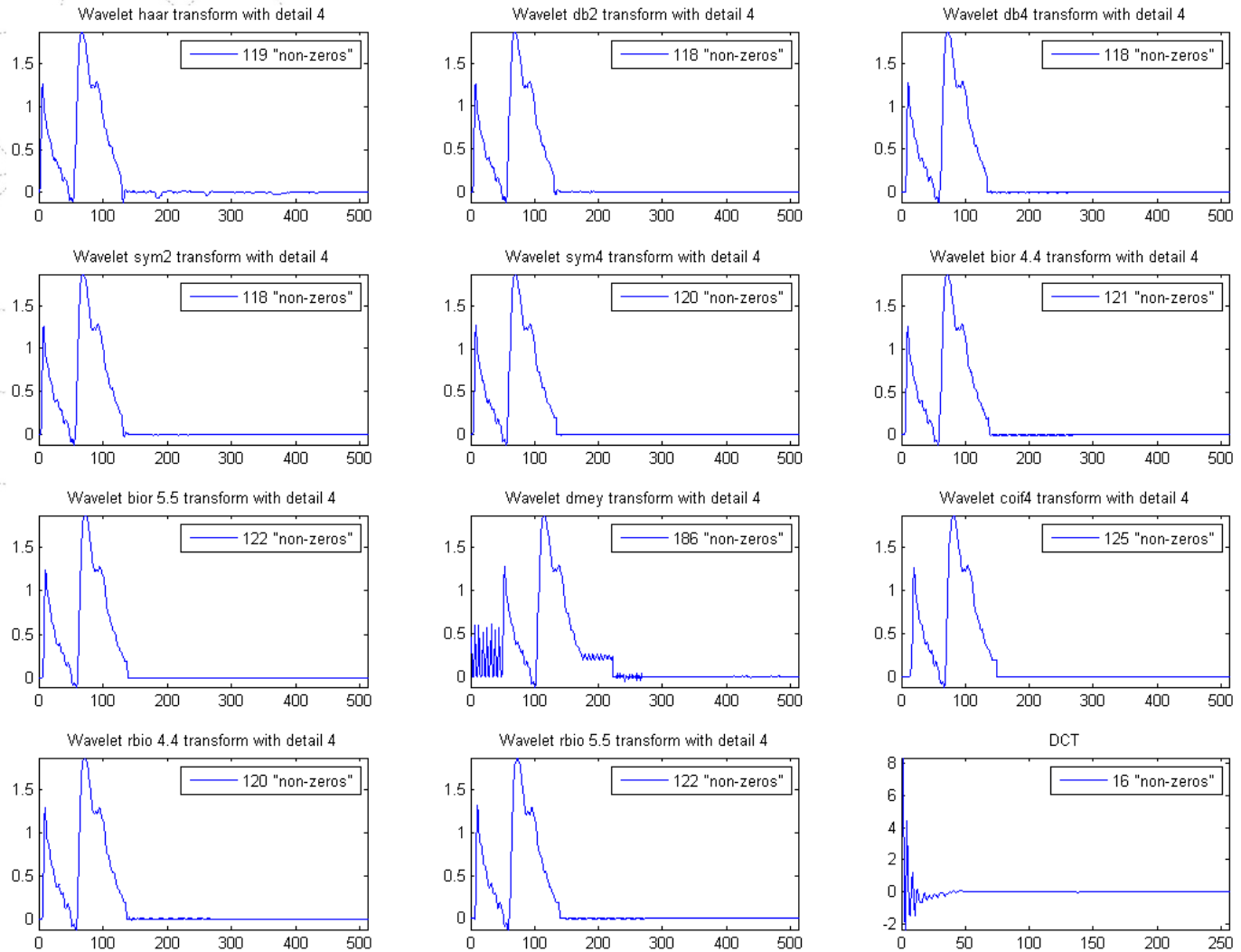
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# PG – K



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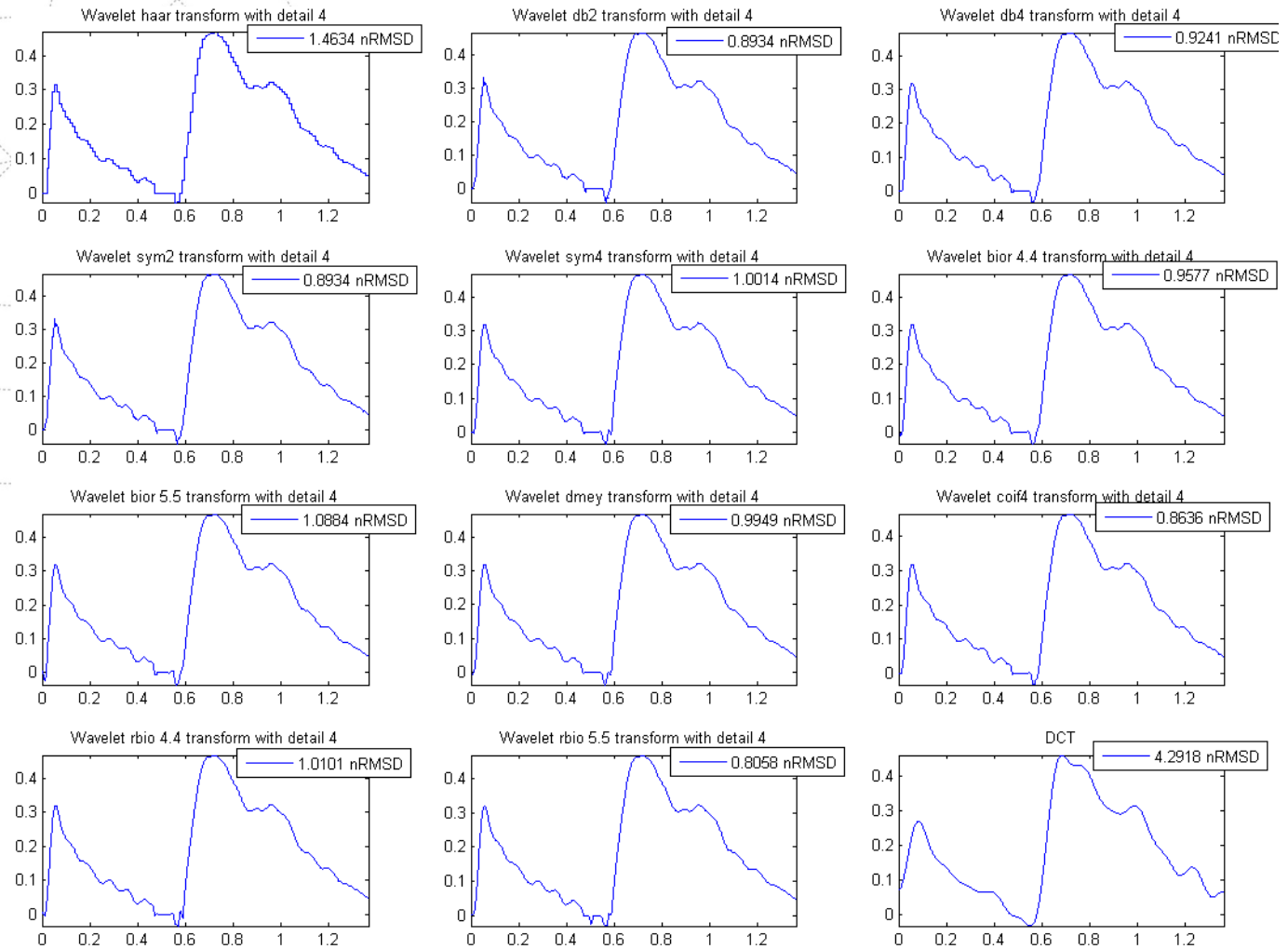
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# PG – rec



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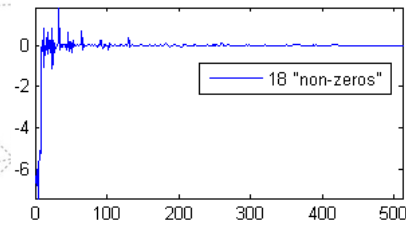
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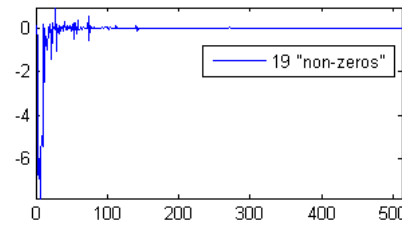


# BCG - K

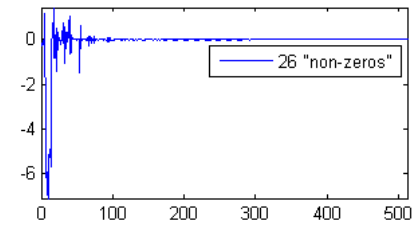
Wavelet haar transform with detail 8



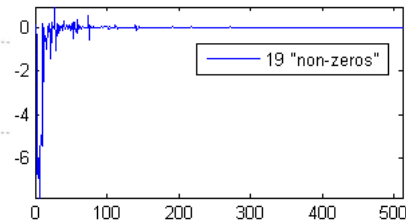
Wavelet db2 transform with detail 8



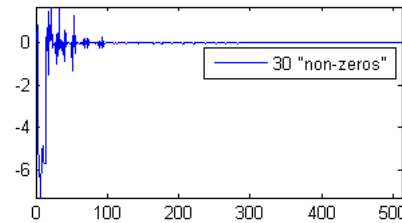
Wavelet db4 transform with detail 8



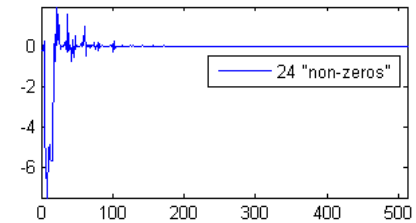
Wavelet sym2 transform with detail 8



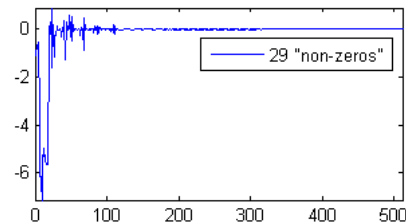
Wavelet sym4 transform with detail 8



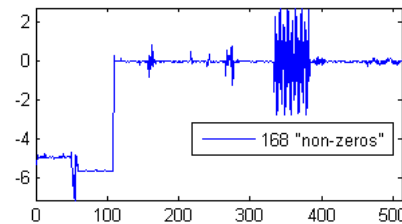
Wavelet bior 4.4 transform with detail 8



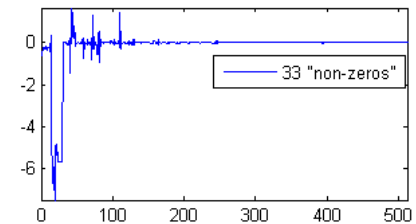
Wavelet bior 5.5 transform with detail 8



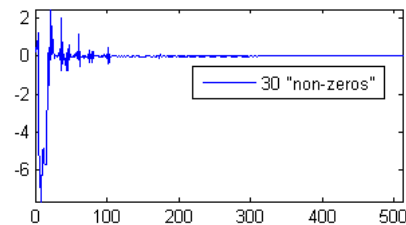
Wavelet dmey transform with detail 8



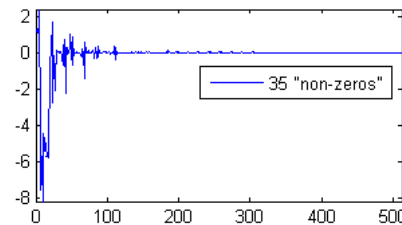
Wavelet coif4 transform with detail 8



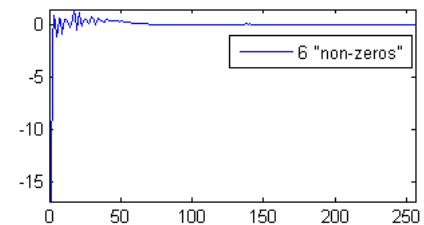
Wavelet rbio 4.4 transform with detail 8



Wavelet rbio 5.5 transform with detail 8



DCT



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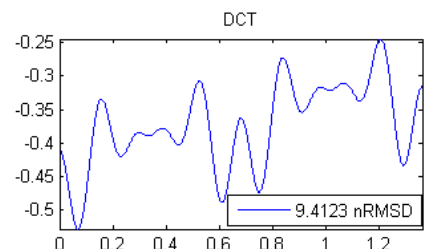
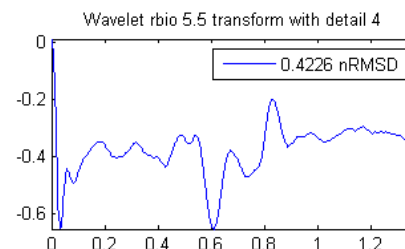
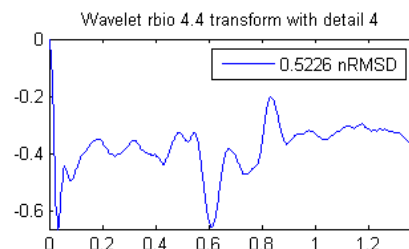
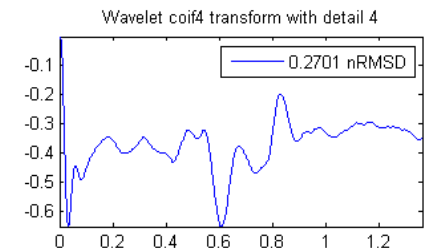
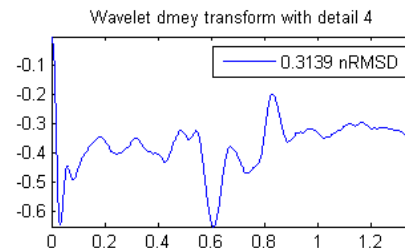
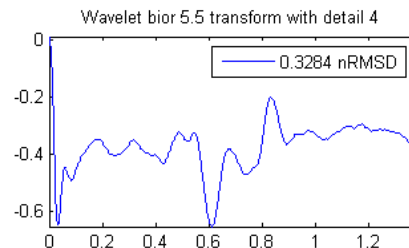
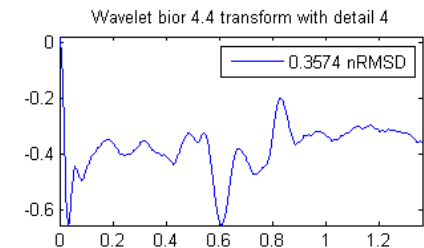
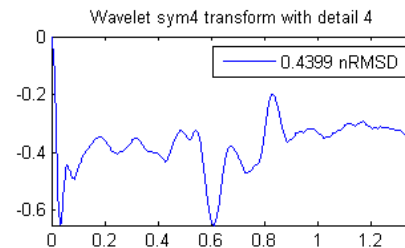
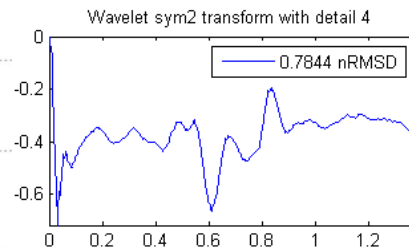
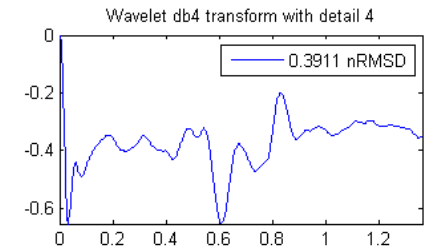
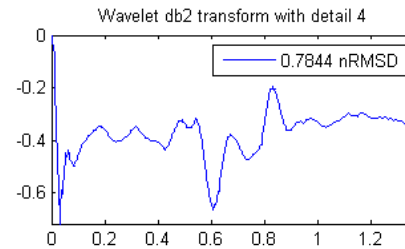
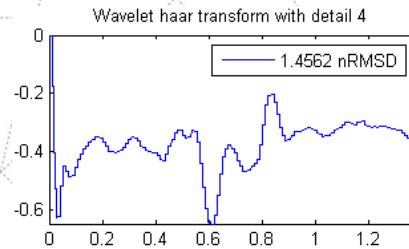
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# BCG – rec



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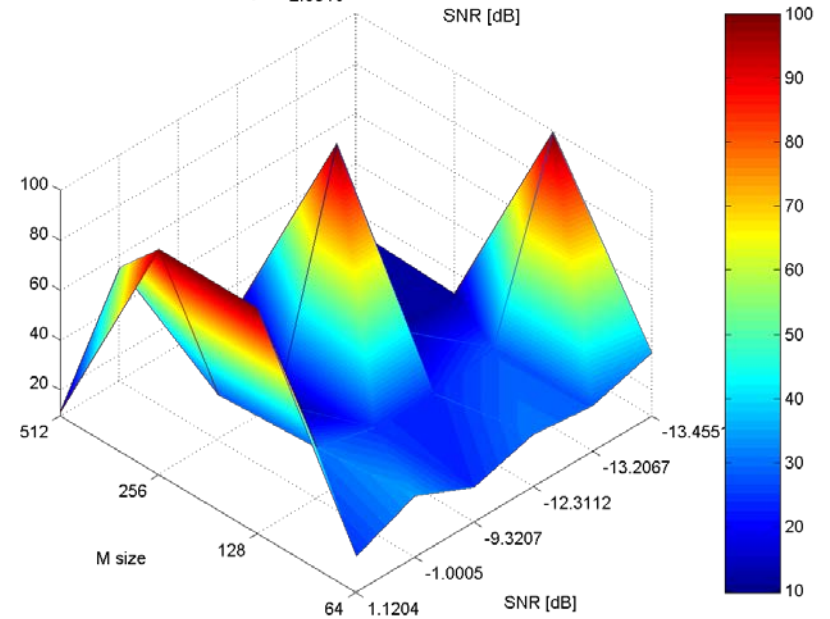
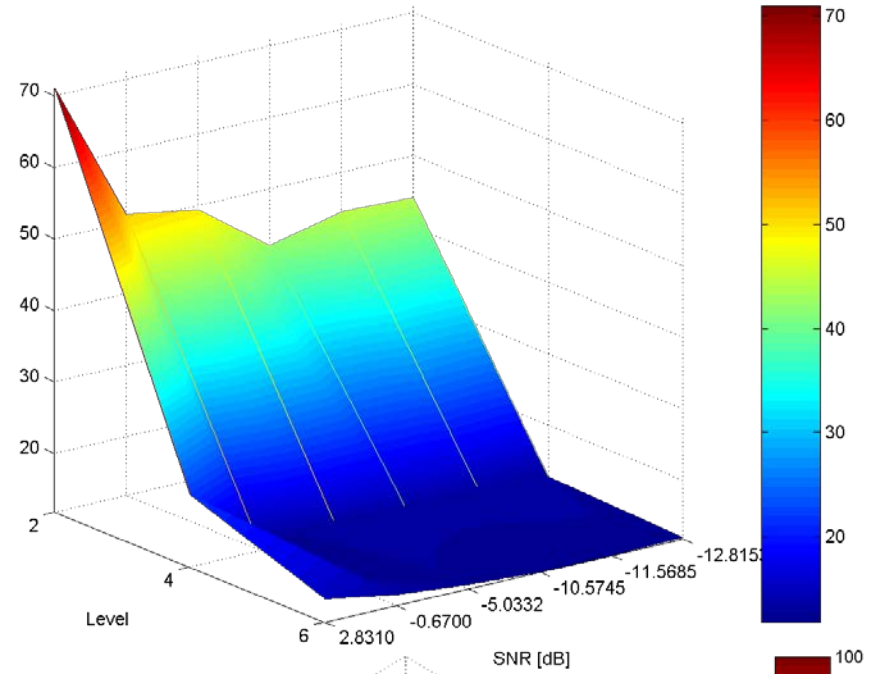
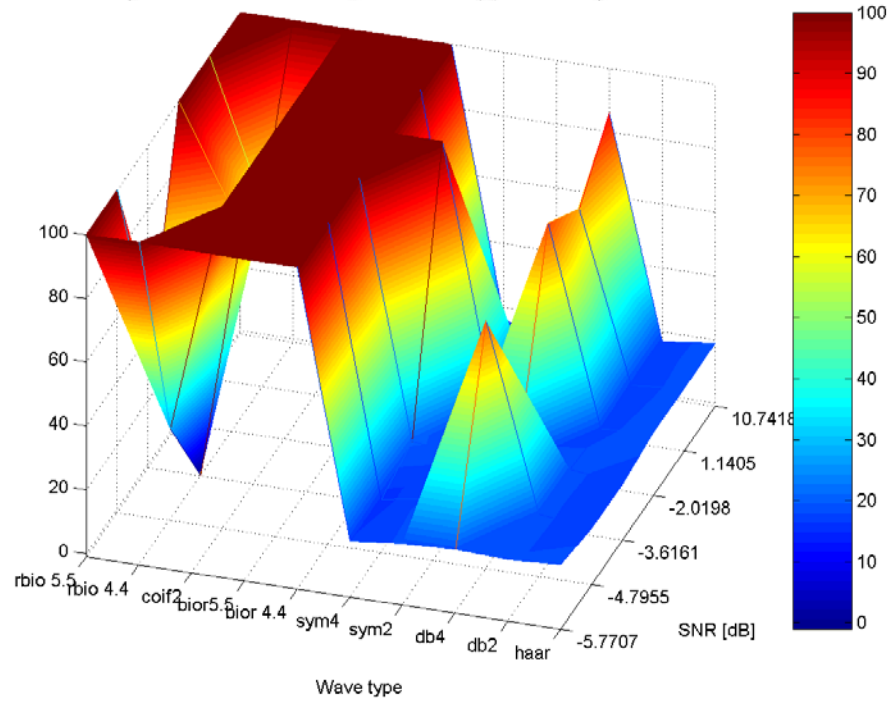


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# Noise?



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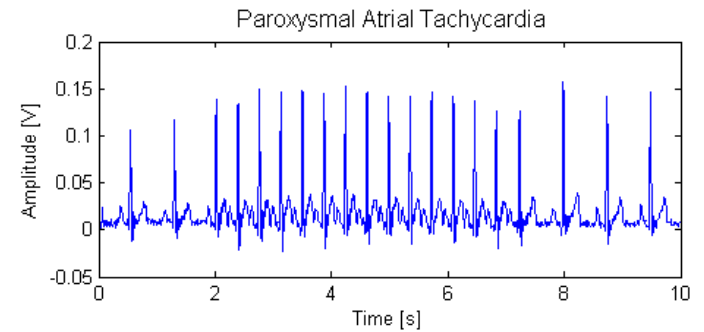
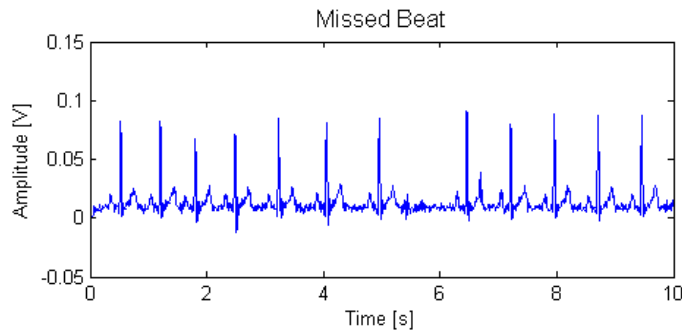
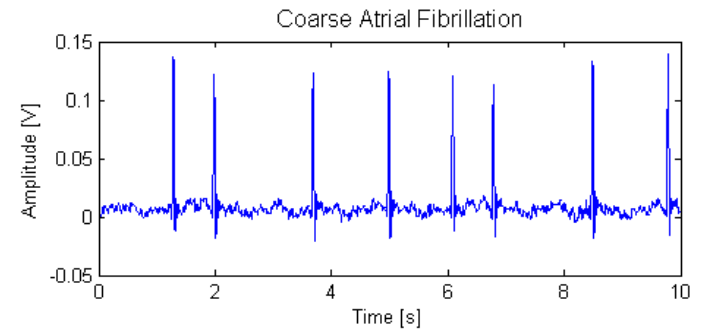
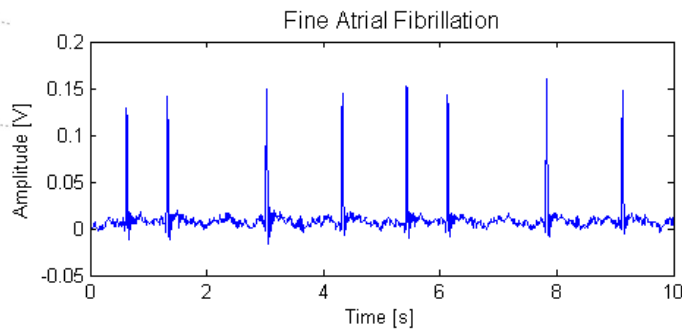
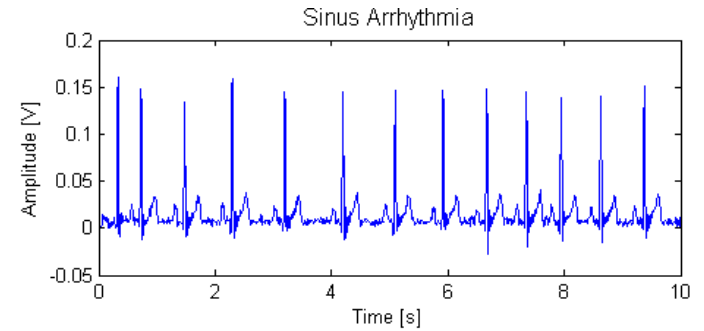
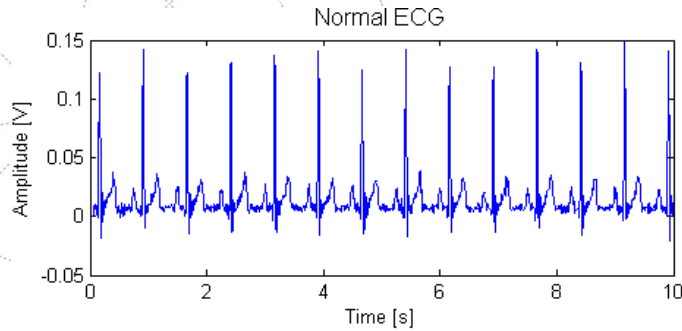


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# Diseases?



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# Networking

Lossy transmissions cannot attain interpolation

Zero mean-signals  $\Rightarrow E[x]=0$

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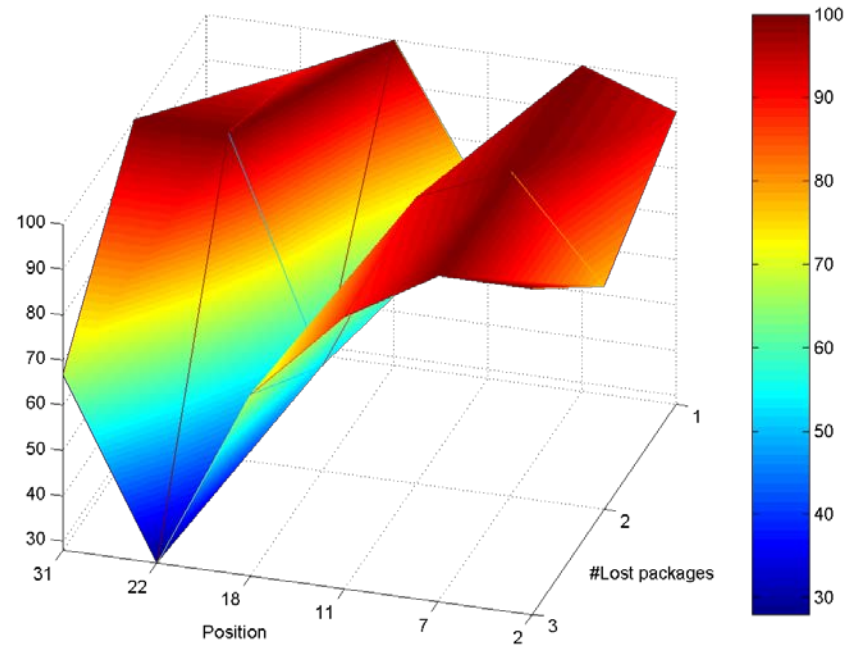
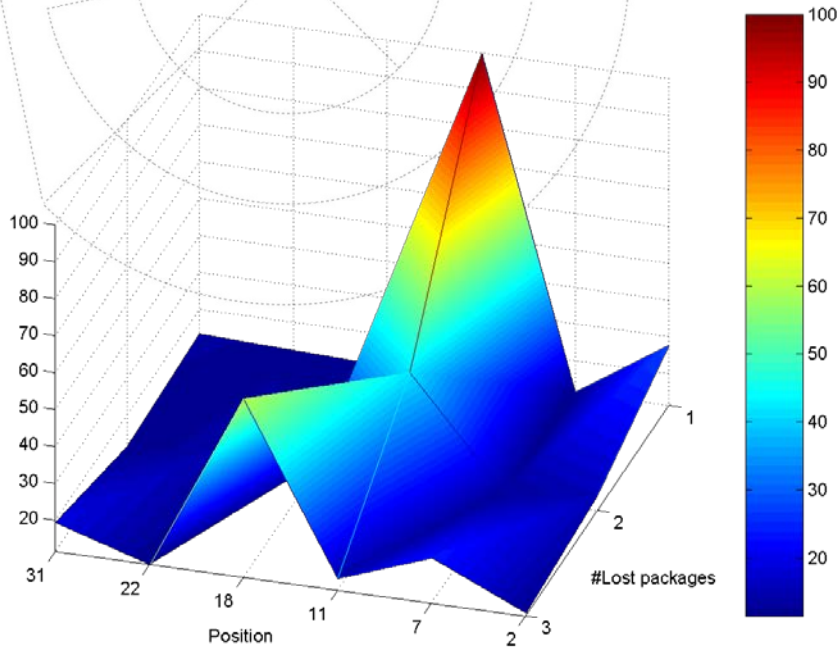


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Samples have variable importance, but hints on their value are vital

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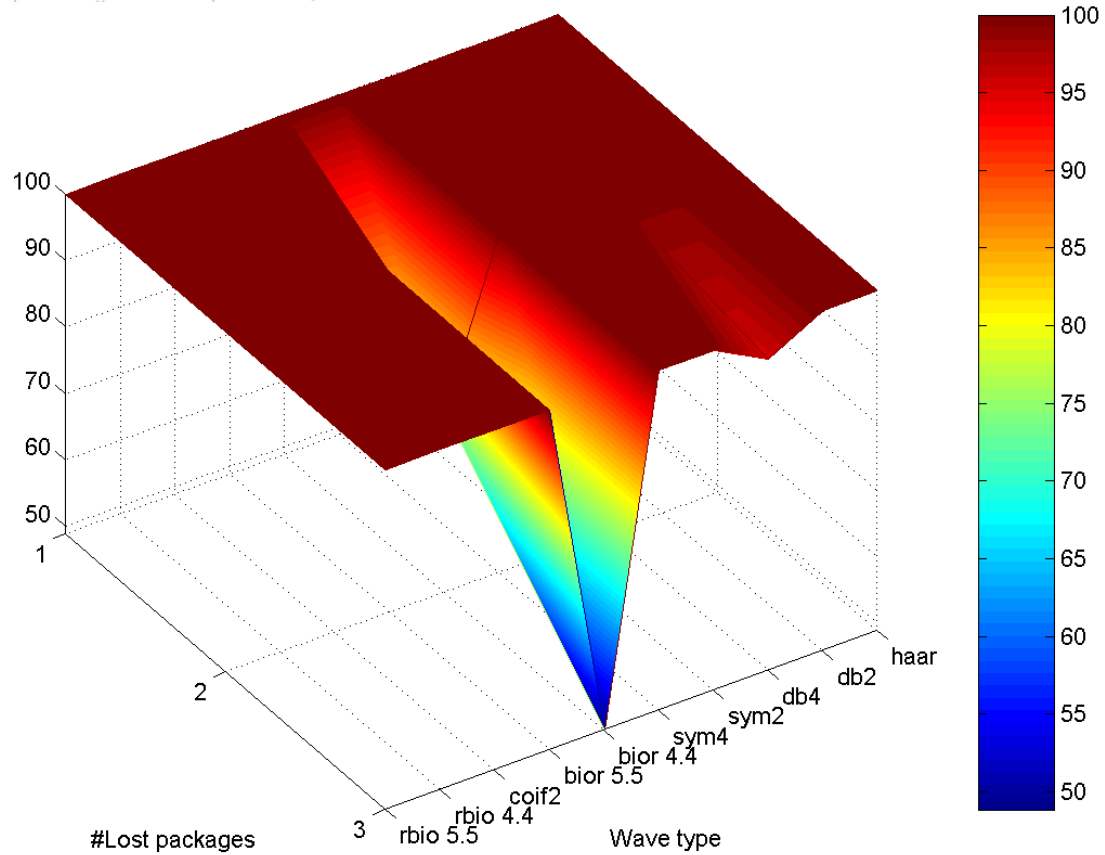


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# Networking



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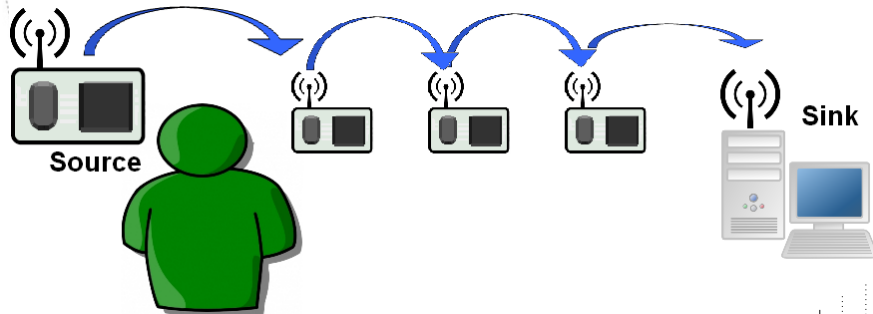
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# Networking

All packets needed, so, if random losses occur, with  $p$  probability, and at most  $k$  attempts, to cross  $h$  hops

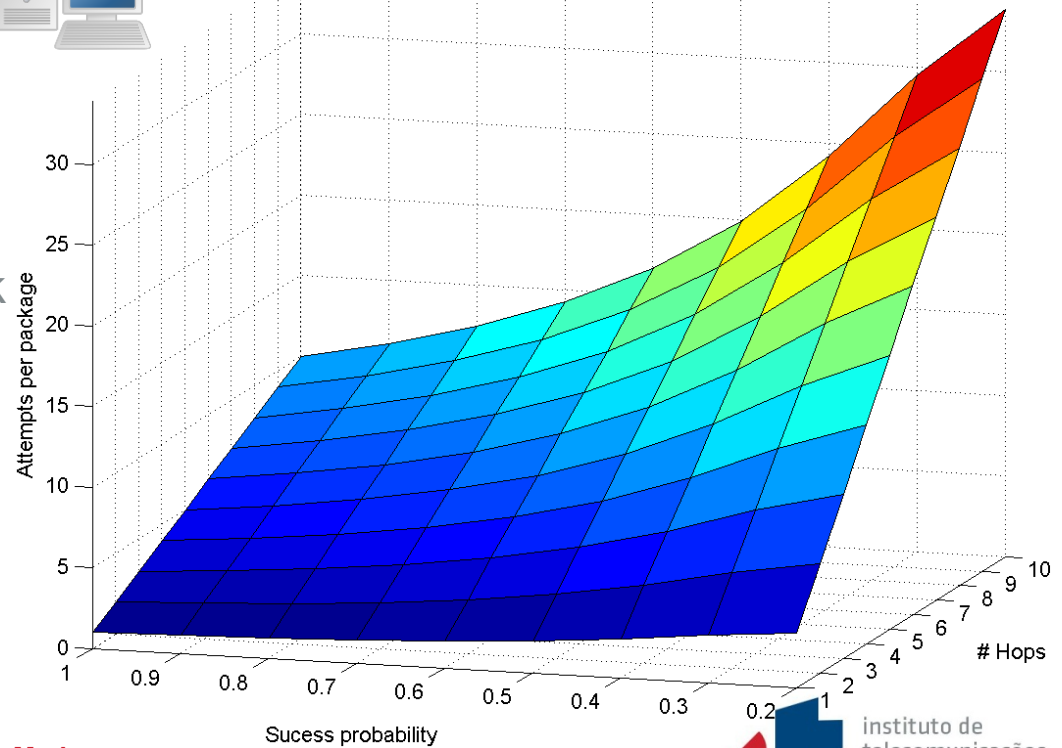


$$\# Att = h \times \sum_{k=1}^{10} kp(1-p)^{k-1}$$

$p=50\%$   
 $k=10$   
 $h=4$

7.9 att/pack

Compression of 8:1...



# Networking

Power consumption in sensor nodes is dominated by radio (10:1 vs MCU only)

Lowering the amount of data to acquire/process reduces MCU power

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Own prototype with general purpose MCU board >0.2 W:  $E_{saved} = 0.233 \frac{N}{M} t$

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Own prototype with general purpose MCU board >0.2 W:  $E_{saved} = 0.233 \frac{N}{M} t$

Use compressed sensing even respecting Nyquist rate, to virtually increase sampling freq

Projection in the sensor allows transmission of colored noise (security)



# Networking

In practical terms...



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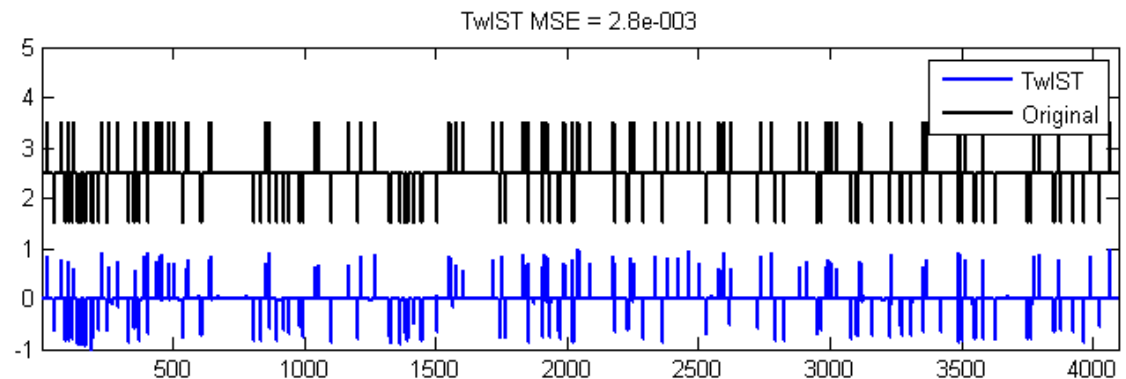
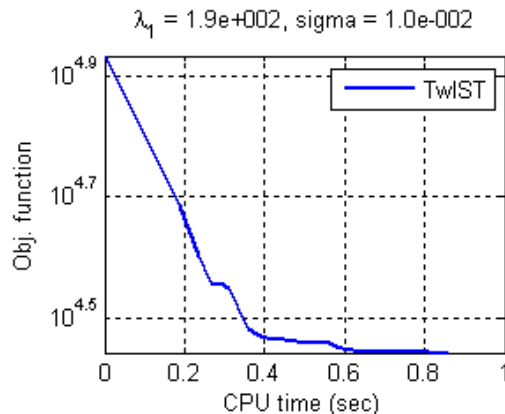
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# Conclusions

1. ECG, PPG, ICG, and BCG are signals with heavy-tailed distributions on several different WT
2. If a restrict percentage of the WT coefficients is accurately recovered ( $K \approx 20$ ), the signal can be reconstructed w/ concern
3. Biorthogonal 4.4 and 5.5 and their respective reverse, and Coiflet offer the worst rec result
4. Symlets 2&4 and Daubechies 2 &4 offer the best reconstructions, although having about the same compressibility
5. Small variations in TwIST regularization parameter ( $\xi$ ) and WT level to be introduced according to the signal

# Conclusions

6. TCP-type protocols are mandatory, no data may be lost
7. Impulsive signals (ECG and BCG) are less penalized than PPG
8. Compression ratios around 8 will be enough to improve real wireless networks lifetime
9. Reconstruction within non-displaying nodes is not profitable
10. There is a bottleneck due to compression representativeness, time of response in emergency, amount of data in each packet, and overall network security



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# Thank You!

And grants of Fundação para a Ciência e Tecnologia :

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